

# EE 435

## Lecture 3

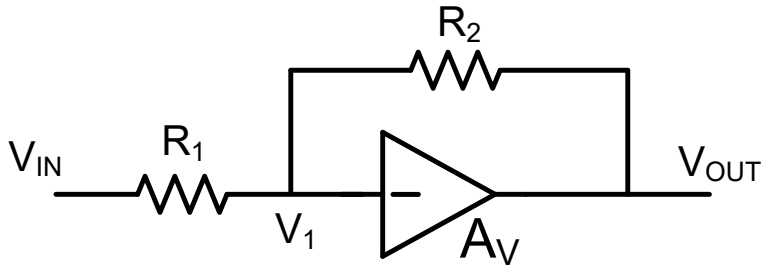
### Design Space Exploration

with applications to single-stage amplifier design

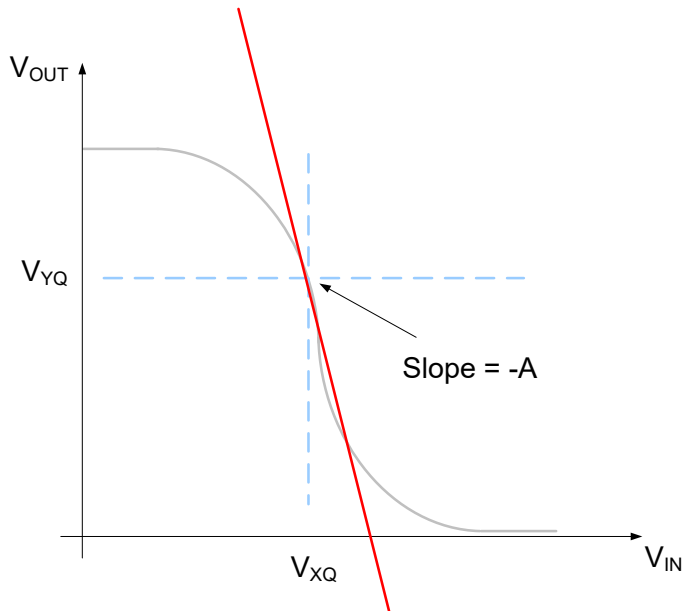
### Systematic Strategies for Designing and Analyzing Op Amps

## Review from last lecture:

# Single-ended Op Amp Inverting Amplifier



$$\left. \begin{aligned} V_O &= (-A)(V_1 - V_{XQ}) + V_{YQ} \\ V_1 &= \frac{R_1}{R_1 + R_2} V_O + \frac{R_2}{R_1 + R_2} V_{IN} \end{aligned} \right\}$$



Summary:

$$V_O = -\frac{R_2}{R_1} V_{iss} + V_{XQ} + \frac{R_2}{R_1} (V_{XQ} - V_{inQ})$$

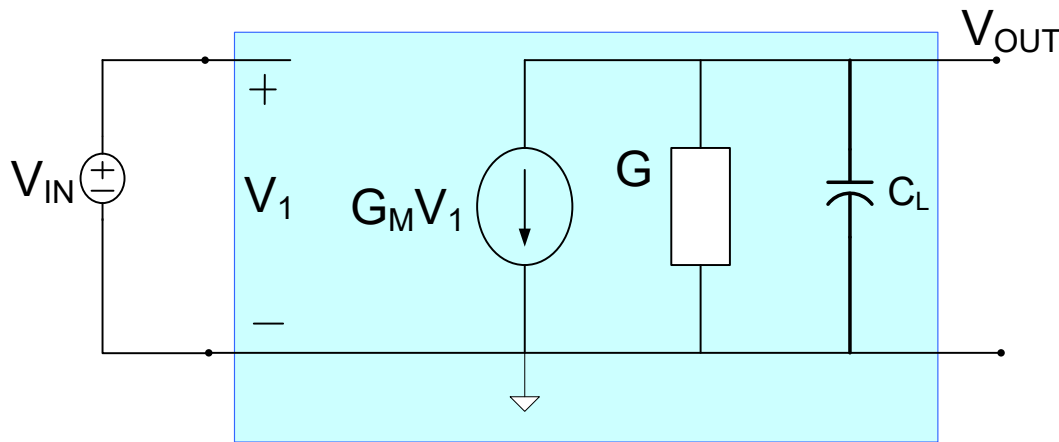
---

What type of circuits have the transfer characteristic shown?

Review from last lecture:

# Single-stage single-input low-gain op amp

(unilateral with infinite input impedance and one capacitor)

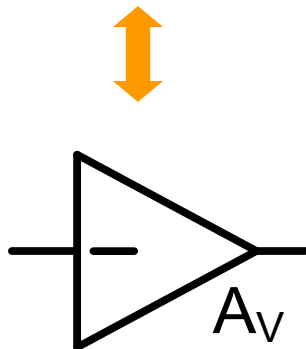


$$A_V = \frac{-G_M}{sC_L + G}$$

$$A_{V0} = \frac{-G_M}{G}$$

$$BW = \frac{G}{C_L}$$

$$GB = \left(\frac{G_M}{G}\right) \left(\frac{G}{C_L}\right) = \frac{G_M}{C_L}$$



**GB and  $A_{V0}$  are two of the most important parameters in an op amp**

## Review from last lecture:

How do we design an amplifier with a given architecture in general or this architecture in particular?

What is the design space?

Generally  $V_{SS}$ ,  $V_{DD}$ ,  $C_L$  (and possibly  $V_{OUTQ}$ ) will be fixed

Must determine  $\{W_1, L_1, I_{DQ} \text{ and } V_{INQ}\}$

Thus there are 4 design variables

But  $W_1$  and  $L_1$  appear as a ratio in almost all performance characteristics of interest

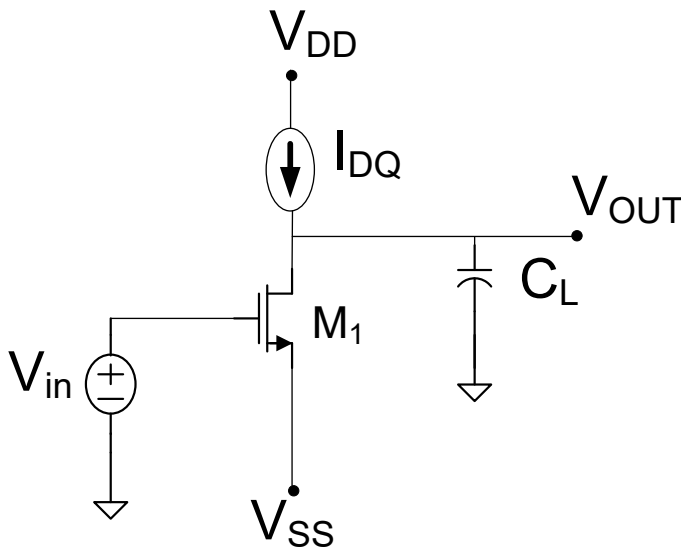
and  $I_{DQ}$  is related to  $V_{INQ}$ ,  $W_1$  and  $L_1$   
(this is a constraint)

Thus the design space generally has only two independent variables or **two degrees of freedom**

$$\left\{ \frac{W_1}{L_1}, I_{DQ} \right\}$$

Thus design or “synthesis” with this architecture involves exploring the two-dimensional design space

$$\left\{ \frac{W_1}{L_1}, I_{DQ} \right\}$$



Review from last lecture:

## How do we design an amplifier with a given architecture ?

1. Determine the design space
2. Identify the constraints
3. Determine the entire set of unknown variables and the Degrees of Freedom
4. Determine an appropriate parameter domain
5. Explore the resultant design space with the identified number of Degrees of Freedom

## Review from last lecture:

# Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain :

$$\{g_m, g_0\}$$

$$A_{V0} = \frac{-g_m}{g_0} \quad GB = \frac{g_m}{C_L}$$

Natural design parameter domain:

$$\left\{ \frac{W}{L}, I_{DQ} \right\}$$

$$A_{V0} = \left[ \frac{\sqrt{2\mu C_{OX}}}{\lambda} \right] \left[ \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}} \right] \quad GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right]$$

Alternate parameter domain:

$$\{P, V_{EB}\}$$

$$A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right]$$

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

Process Dependent

## Review from last lecture:

# Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain :  $\{g_m, g_0\}$

$$A_{v0} = \frac{-g_m}{g_0} \qquad GB = \frac{g_m}{C_L}$$

Natural design parameter domain:  $\left\{ \frac{W}{L}, I_{DQ} \right\}$

$$A_{v0} = \left[ \frac{\sqrt{2\mu C_{OX}}}{\lambda} \right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}} \qquad GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}}$$

Alternate parameter domain:

$\{P, V_{EB}\}$

$$A_{v0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right] \qquad GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

Architecture Dependent

## Review from last lecture:

# Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain :

$$\{g_m, g_0\}$$

$$A_{V0} = \frac{-g_m}{g_0}$$

$$GB = \frac{g_m}{C_L}$$

Natural design parameter domain:

$$\left\{ \frac{W}{L}, I_{DQ} \right\}$$

$$A_{V0} = \left[ \frac{\sqrt{2\mu C_{OX}}}{\lambda} \right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}}$$

$$GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}}$$

Alternate parameter domain:

$$\{P, V_{EB}\}$$

$$A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right]$$

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

- Alternate parameter domain gives considerable insight into design
- Easy to map from alternate parameter domain to natural parameter domain
- Alternate parameter domain provides modest parameter decoupling
- $A_{V0} \left[ \frac{\lambda}{2} \right]$  and  $GB \left[ \frac{V_{DD} C_L}{2} \right]$  figures of merit for comparing architectures



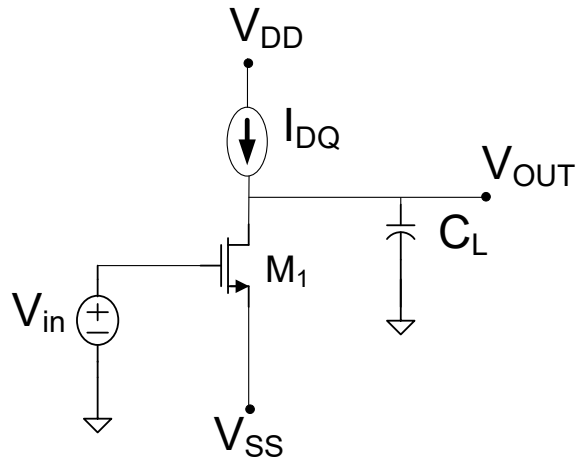
## Review from last lecture:

# Design With the Basic Amplifier Structure

Consider basic op amp structure

**Alternate parameter domain:**  $\{P, V_{EB}\}$

**Degrees of Freedom: 2**



$$A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right]$$

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

$$I_{DQ} = \frac{P}{V_{DD}}$$

$$\frac{W}{L} = \frac{P}{V_{DD} \mu C_{OX} V_{EB}^2}$$

$$V_{INQ} = V_{SS} + V_T + \sqrt{I_{DQ} \frac{2}{\mu C_{OX}} \frac{L}{W}}$$

---

But what if the design requirement dictates that  $V_{INQ}=0$ ?

Question: How can one meet two or more performance requirements with one design degree of freedom with this circuit?

**Degrees of Freedom: 1**

Luck or Can't

# Key Observation about Analog Circuit Design

- **Matching of R's, C's and MOSFETs and BJTs can be very good**

  - .1%, .01%, or even better

  - Good layout is critical for good matching

  - Large area often need for good ratio accuracy

  - Good matching can be maintained over wide temperature range

- **Absolute value of R's, C's, and MOS and BJT parameters very bad**

  - +/- 30% or worse

  - Termed Process/Voltage/Temperature (PVT) effects

  - Values can be highly temperature dependent

  - Several % variation over 100°C range or worse

- **Trimming/Calibration at production test can compensate parameter variations**

  - Adds some cost for test

  - Trim elements consume considerable area

  - Temperature dependence still a challenge

# How do we design an amplifier with a given architecture ?

1. Determine the design space
2. Identify the constraints
3. Determine the entire set of unknown variables and the Degrees of Freedom
4. Determine an appropriate parameter domain



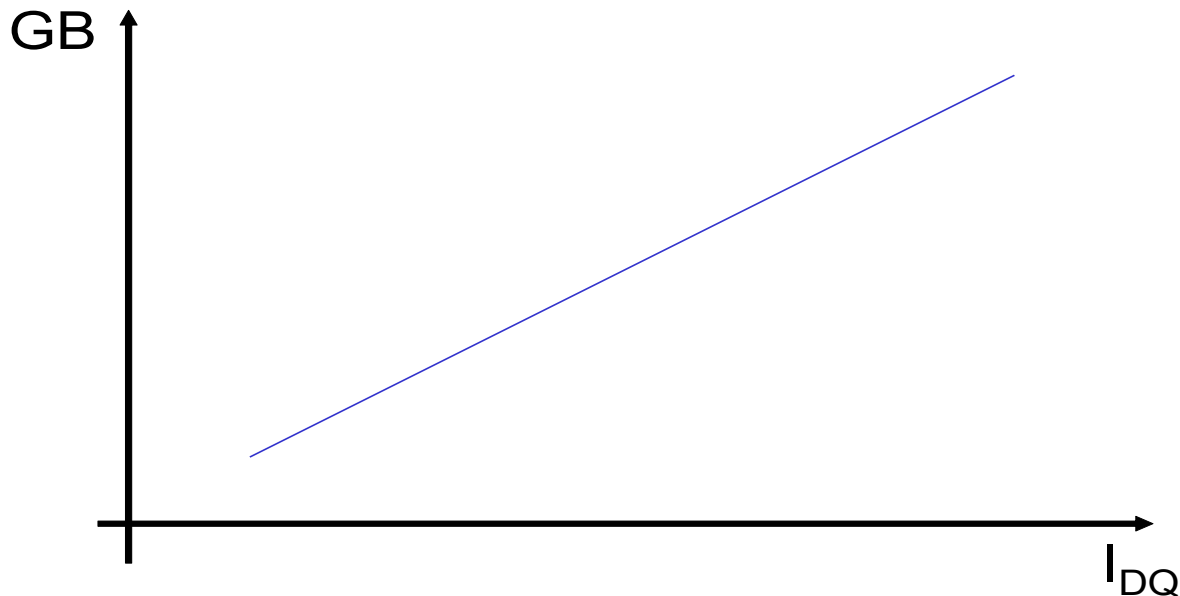
5. Explore the resultant design space with the identified number of Degrees of Freedom

# Design Space Exploration

Question: How does the GB of the single-stage amplifier common-source amplifier change with bias current?

$$GB = \frac{g_m}{C_L} \quad \longrightarrow \quad GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right]$$

GB increases linearly with  $I_{DQ}$

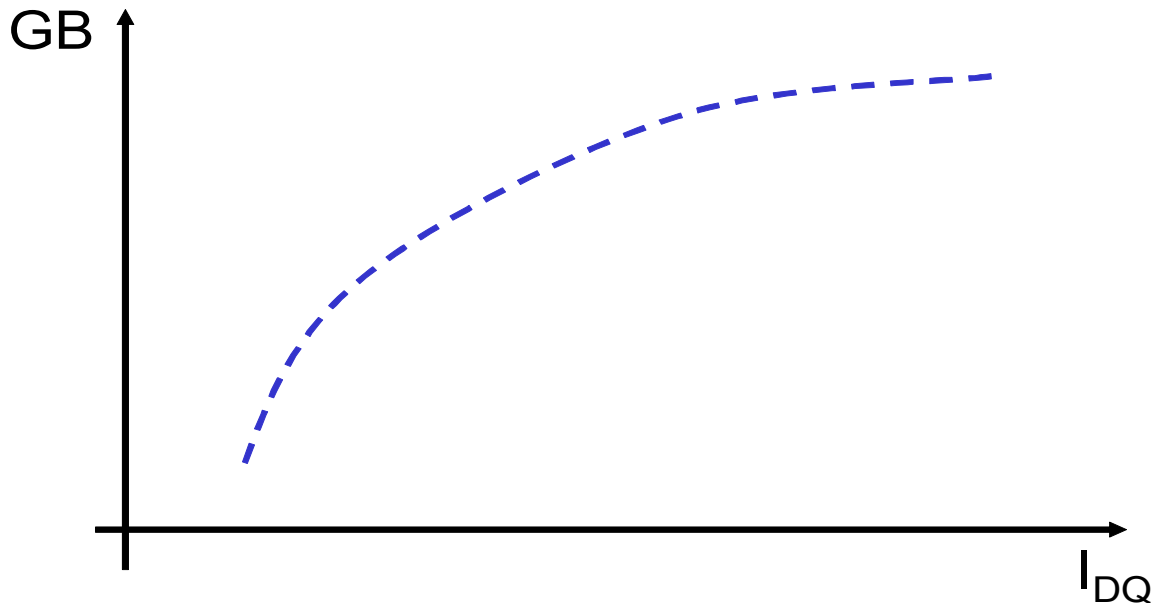


# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right]$$

GB increases with the square root of  $I_{DQ}$

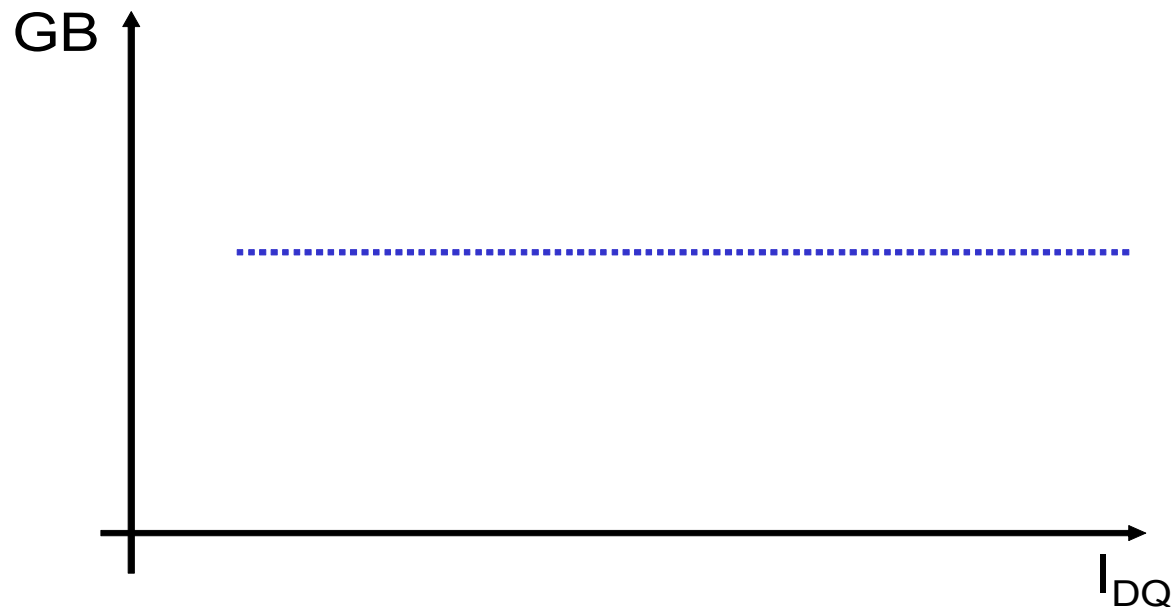


# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

GB independent of  $I_{DQ}$

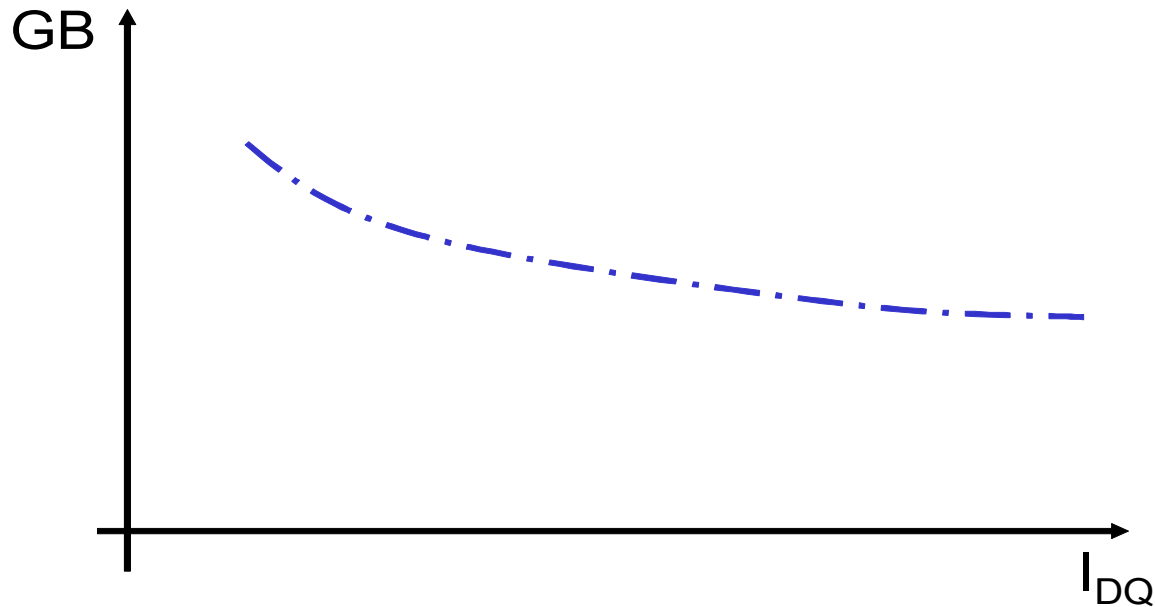


# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{OX} W}{L}}$$

GB decreases with the reciprocal of the square root of  $I_{DQ}$

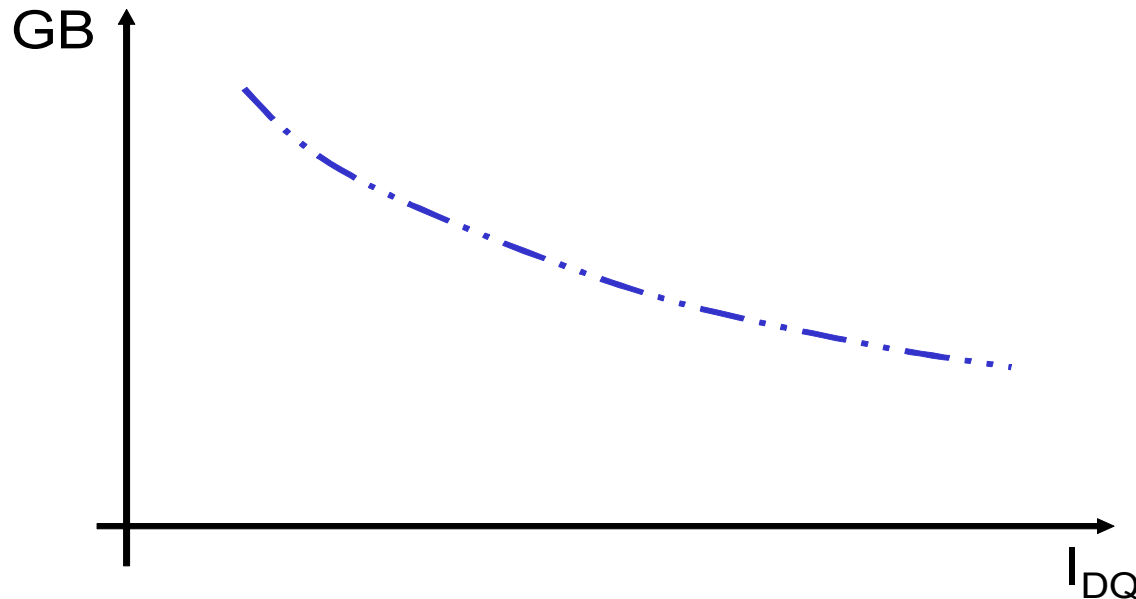


# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{\sqrt{\frac{2\mu C_{OX} WP^3}{LV_{DD}}}}{I_{DQ} C_L}$$

GB decreases with the reciprocal of  $I_{DQ}$





# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right] \quad \longrightarrow \quad \text{Increases Linearly}$$

$$GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right] \quad \longrightarrow \quad \text{Increases Quadratically}$$

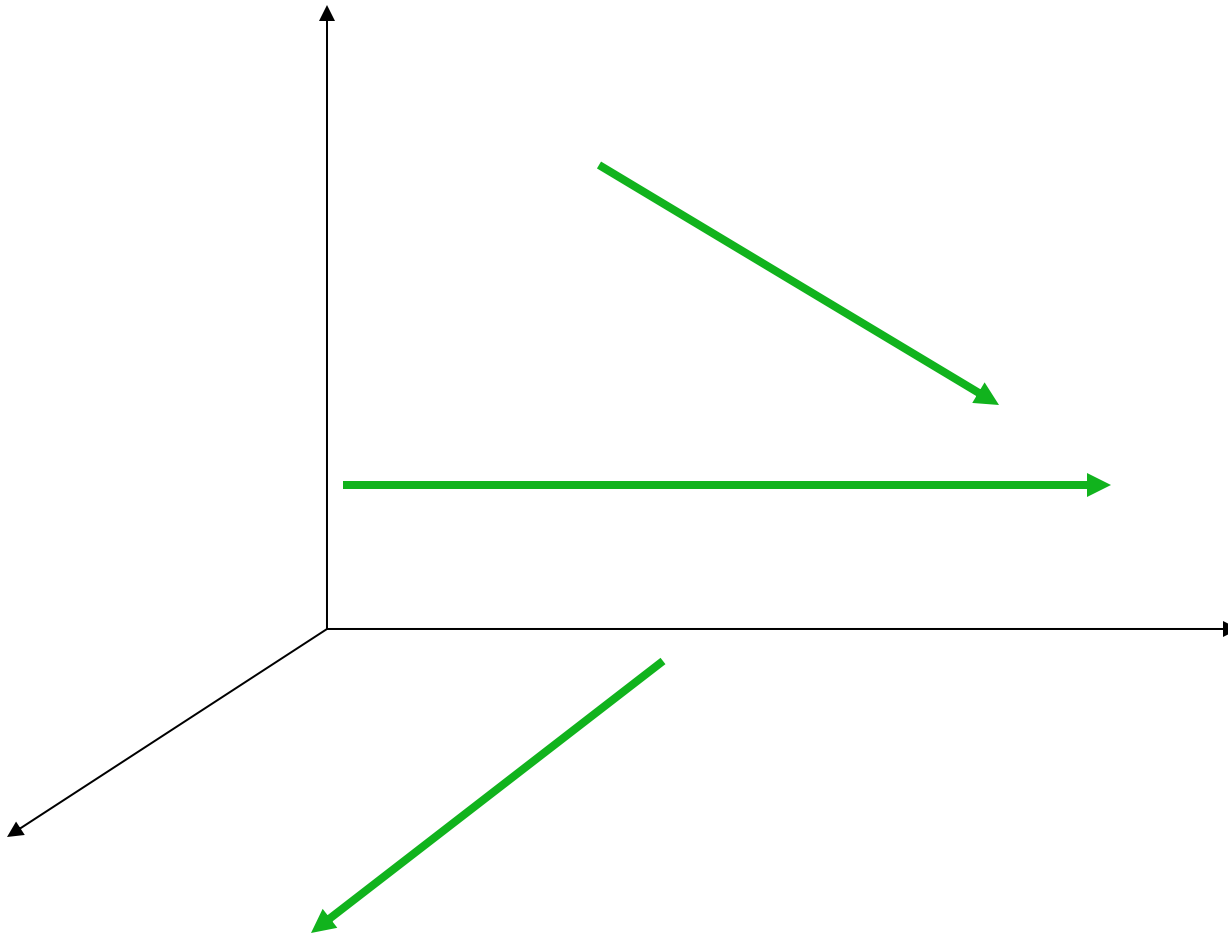
$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right] \quad \longrightarrow \quad \text{Independent of } I_{DQ}$$

$$GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{OX} W}{L}} \quad \longrightarrow \quad \text{Decreases Inverse Quadraticly}$$

$$GB = \frac{\sqrt{\frac{2\mu C_{OX} W P^3}{L V_{DD}}}}{I_{DQ} C_L} \quad \longrightarrow \quad \text{Decreases Inverse Linearly}$$

It depends upon how the design space is explored !!!

# Design Space Exploration



Different trajectories through a design space

# Design Space Exploration

Issue becomes more involved for amplifiers or circuits with more than one transistor

Choice of design parameters can have major impact on insight into design

Size of parameter domain should agree with the number of degrees of freedom

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored

Small signal and natural parameter domains give little insight into design or Performance of operational amplifiers

Recall:

# Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain :

$$\{g_m, g_0\}$$

$$A_{V0} = \frac{-g_m}{g_0}$$

$$GB = \frac{g_m}{C_L}$$

Natural design parameter domain:

$$\left\{ \frac{W}{L}, I_{DQ} \right\}$$

$$A_{V0} = \left[ \frac{\sqrt{2\mu C_{OX}}}{\lambda} \right] \frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}}$$

$$GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}}$$

Alternate parameter domain:

$$\{P, V_{EB}\}$$

$$A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right]$$

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

# Design Space Exploration

Question: How does the  $A_{V0}$  of the single-stage amplifier change with  $V_{EB}$ ?

$$A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right]$$

$A_{V0}$  decreases with the reciprocal of  $V_{EB}$

Even though there are 2 degrees of freedom, the dependence of  $V_{EB}$  on  $A_{V0}$  is unambiguous

Question: How does the  $A_{V0}$  of the single-stage amplifier change with  $P$ ?

$A_{V0}$  is independent of  $P$  if  $V_{EB}$  is fixed

Even though there are 2 degrees of freedom, the dependence of  $P$  on  $A_{V0}$  is unambiguous provided  $V_{EB}$  is fixed

# Design Space Exploration

Question: How does the GB of the single-stage amplifier change with P?

$$GB = \left[ \frac{2}{V_{DD} C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$

GB increases linearly with P ?

This is essentially the same question of how GB varies with  $I_{DQ}$

Answer depends on how  $V_{EB}$  changes

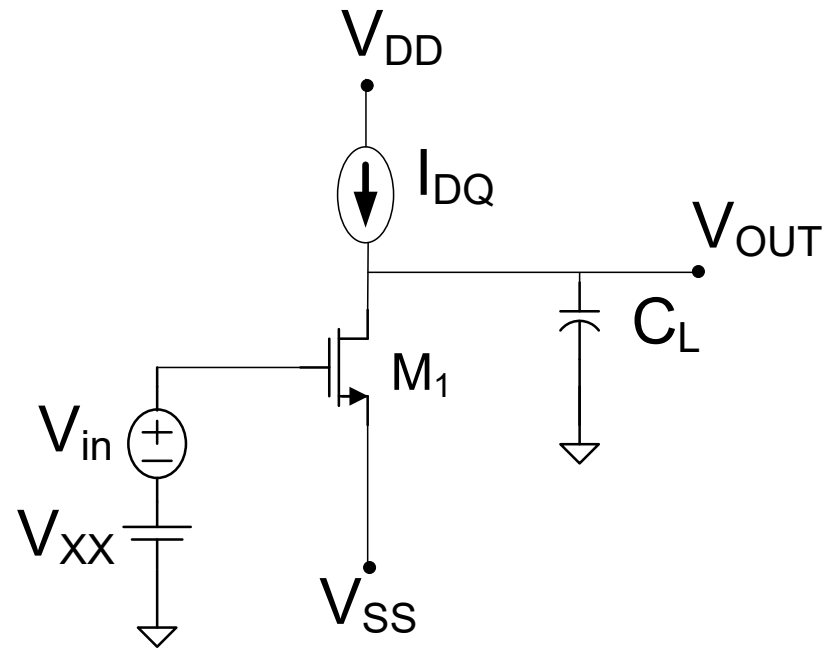
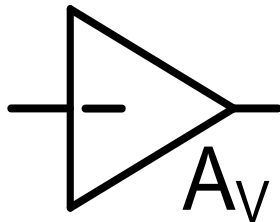
GB increases linearly with P provided the  $A_{V0}$  is fixed ?

This answer is unambiguous since fixing  $A_{V0}$  fixes  $V_{EB}$

Often in situations where the dc gain requirements are fixed and this necessitates a very unfavorable tradeoff between GB and power in this structure since P is a critical “resource” in most applications !

# Single-Stage Low-Gain Op Amps

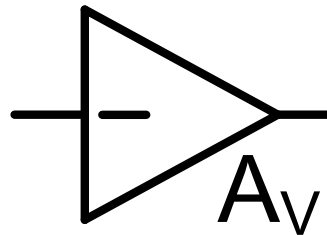
- Single-ended input



Basic single-stage op amp

# Single-Stage Low-Gain Op Amps

- Single-ended input



## Observations:

- This circuit often known as a common source amplifier
- Gain in the 30dB to 45dB range
- Inherently a transconductance amplifier since output impedance is high
- Voltage gain is ratio of transconductance gain to output conductance
- Critical to know degrees of freedom in design and know how to systematically explore design space
- Alternative parameter domain much more useful for design than small-signal domain or natural domain
- **Performance of differential circuits will be obtained by inspection from those of the single-ended structures**

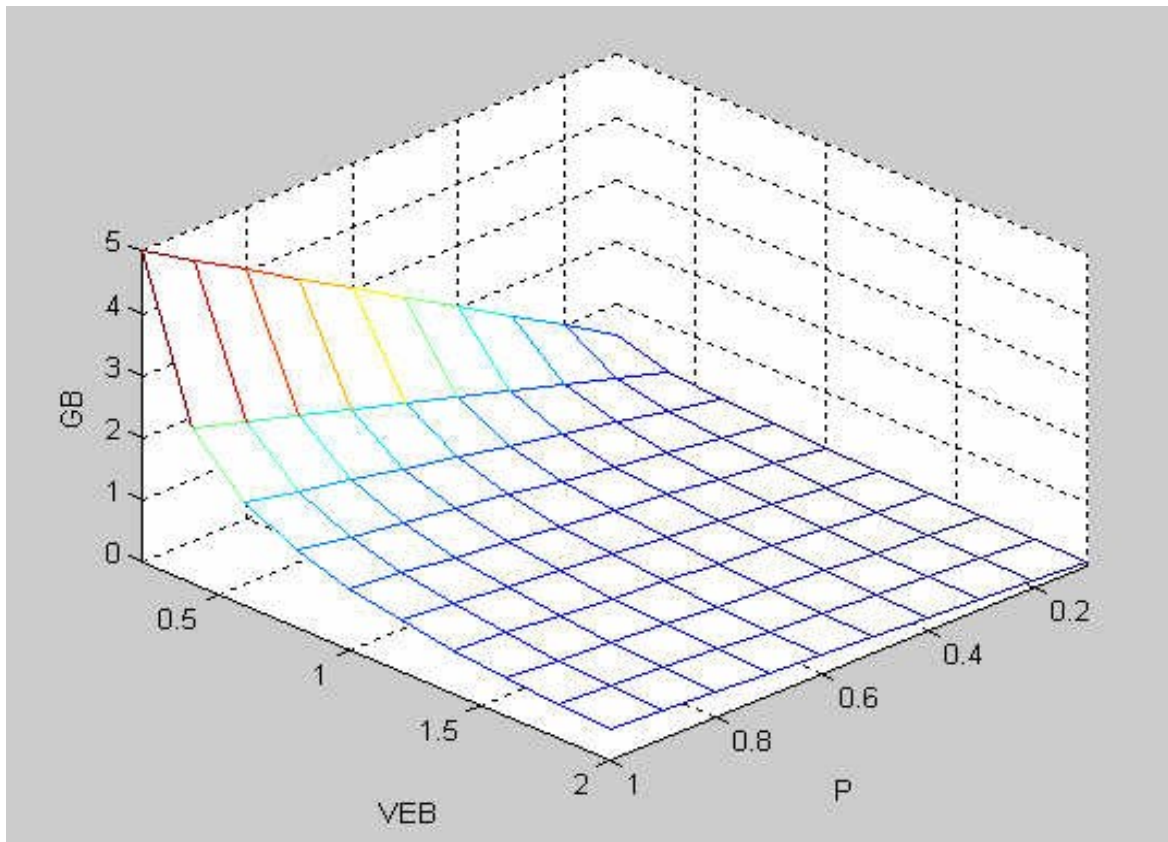


## Review

- Design space is often a high-dimensional system with many local extrema (minimums or maximums)
- Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored

## Design Space for Single-Stage Op Amp

$$GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$



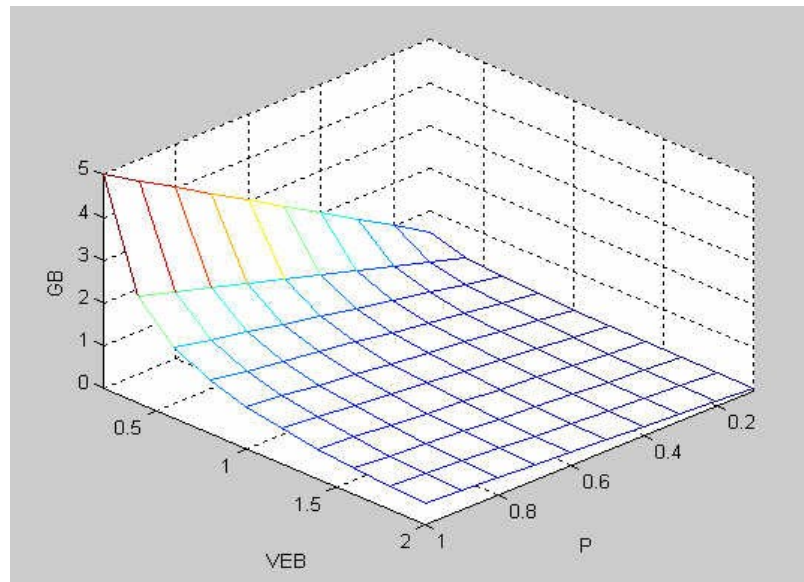
Can we say that GB increases linearly with P ?

# Review

- Multiple parameter domains can be used to characterize and explore a design space
- Performance characteristics of interest take on many different forms depending upon how design space is characterized
- Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
- Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
  - Facilitates comparison of different architectures
- Choice of characterization parameters can make a major difference on how hard it is to explore a design space

## Design Space for Single-Stage Op Amp

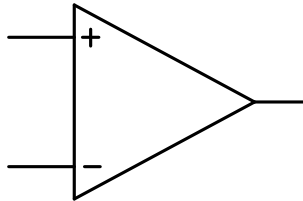
$$GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right]$$



Can we say that GB increases linearly with P ?

Can we say that GB increases linearly with P if  $A_V$  is fixed?

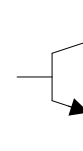
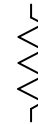
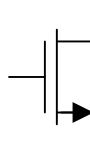
# Noise



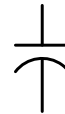
Noise in electronic devices and components introduce noise in electronic systems

Noise is of major concern in amplifiers, ADCs, and DACs

Devices that contribute noise :



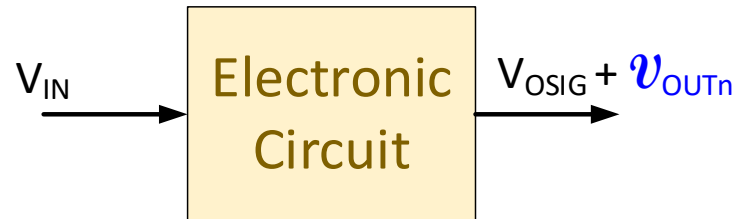
Capacitors and Inductors are noiseless:



# Noise in Electronic Circuits

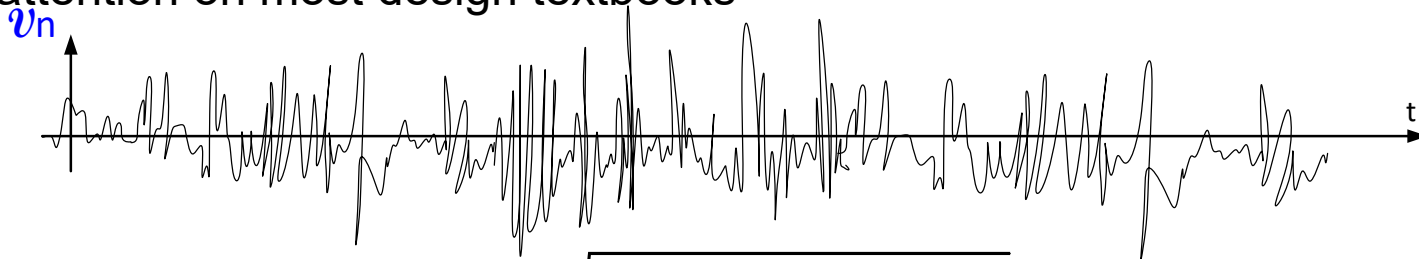
Resistors and transistors contribute device noise

Typically interested in RMS value of the noise voltage on Output of a Circuit and how this compares to the RMS signal on Output



Dynamic Range (which will be defined later) is the ratio of the signal voltage to the noise voltage

Dynamic Range is a Key Property of most systems though often is given little attention on most design textbooks



$$v_{\text{RMS}} = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T v_n^2(t) dt}$$

- Noise be in mV range but could be much smaller or much larger !
- Ratio of RMS signal to RMS noise might be in 1000:1 range but could be much smaller or much larger

# Noise in Electronic Circuits

Resistors and transistors contribute device noise

Noise can be characterized in either the time domain or the spectral (frequency) domain

$$\mathbf{v}_n(t) \longleftrightarrow \mathbf{S}(f)$$

Time domain:

$$\mathbf{v}_{RMS} = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{t=0}^T \mathbf{v}_n^2(t) dt}$$

Difficult to analytically obtain directly !

Frequency domain:

$$\tilde{\mathbf{v}}_{RMS} = \sqrt{\int_{f=0}^{\infty} \mathbf{S}(f) df}$$

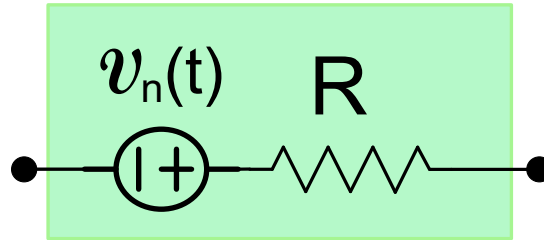
Often much easier to obtain analytically !

It can be shown that:  $\tilde{\mathbf{v}}_{RMS} = \mathbf{v}_{RMS}$

# Noise in Resistors

Resistors and transistors contribute device noise but  
what about charge redistribution DACs ?

Noise in resistors:



Noise spectral density of  $v_n(t)$  at all frequencies for a resistor  $S = 4kTR$

k: Boltzmann's Constant

T: Temperature in Kelvin

$$k = 1.38064852 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$$

$$\text{At } 300\text{K}, kT = 4.14 \times 10^{-21}$$

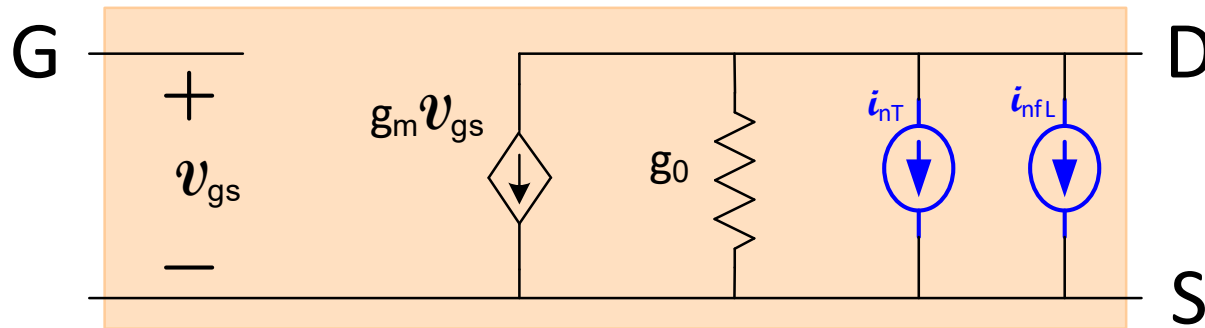
This is termed white noise because  $S$  is independent of  $f$  !



# Noise in Resistors

Resistors and transistors contribute device noise

## Noise in MOS transistors



Noise in saturation region dominated by two noise sources, one thermal noise and other flicker noise (sometimes termed 1/f noise)

Noise spectral density of  $\mathcal{J}_{nT}(t)$  at all frequencies  $S_T = \frac{8kT}{3} g_m$

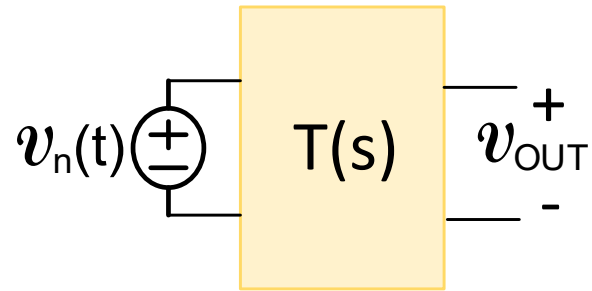
k: Boltzmann's Constant

Noise spectral density of  $\mathcal{J}_{FL}(t)$ ,  $S_{FL}$ , proportional to 1/f T: Temperature in Kelvin

Thermal noise is also white noise because S is independent of f !

Flicker noise termed "colored" or sometimes "pink"

# Noise in Electronic Circuits



Due to any noise voltage source:

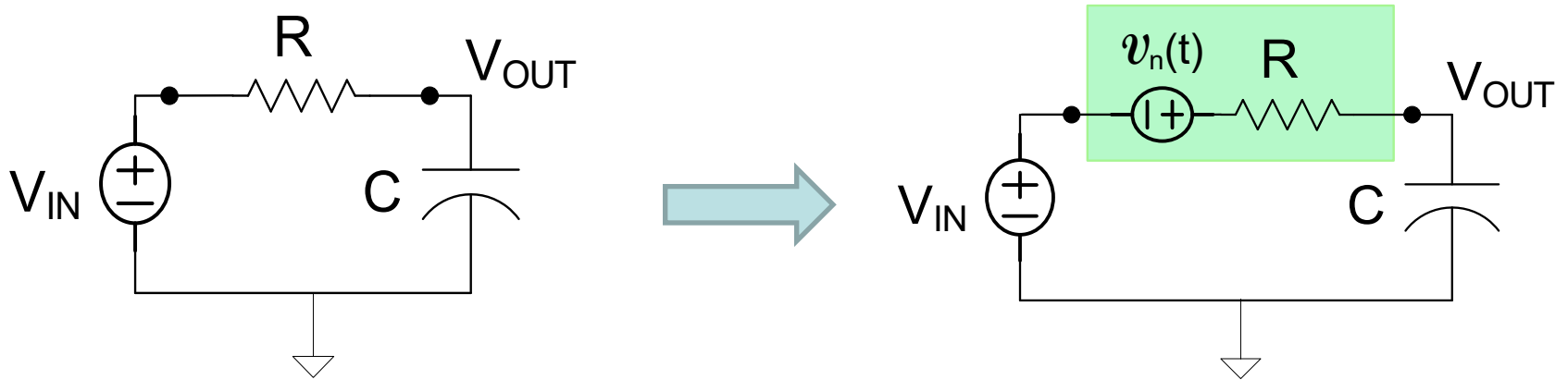
$$S_{V_{OUT}} = S_{V_n} |T_n(j\omega)|^2$$

$T(s)$  is the transfer function from a noise source (current or voltage) to the output

Thus:

$$v_{OUT_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}} df} = \sqrt{\int_{f=0}^{\infty} S_{V_n} |T_n(j\omega)|^2 df}$$

# Example: First-Order RC Network

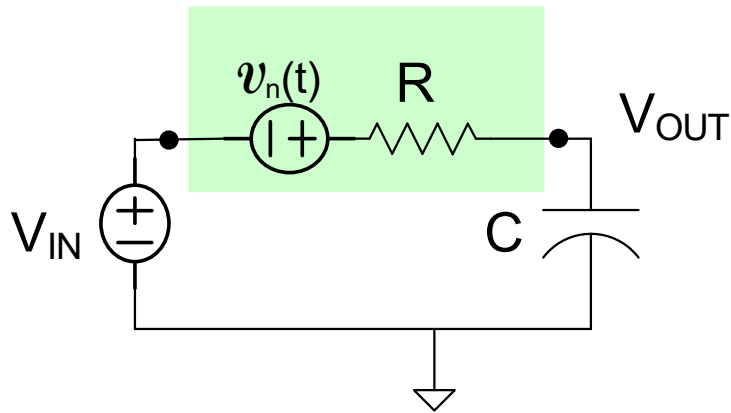


Noise transfer function:  $T_n(s) = \frac{1}{1+RCs}$

$$S_{VOUT} = 4kTR \left( \frac{1}{1+(RC\omega)^2} \right)$$

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{VOUT} df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2 R^2 C^2} df}$$

# Example: First-Order RC Network



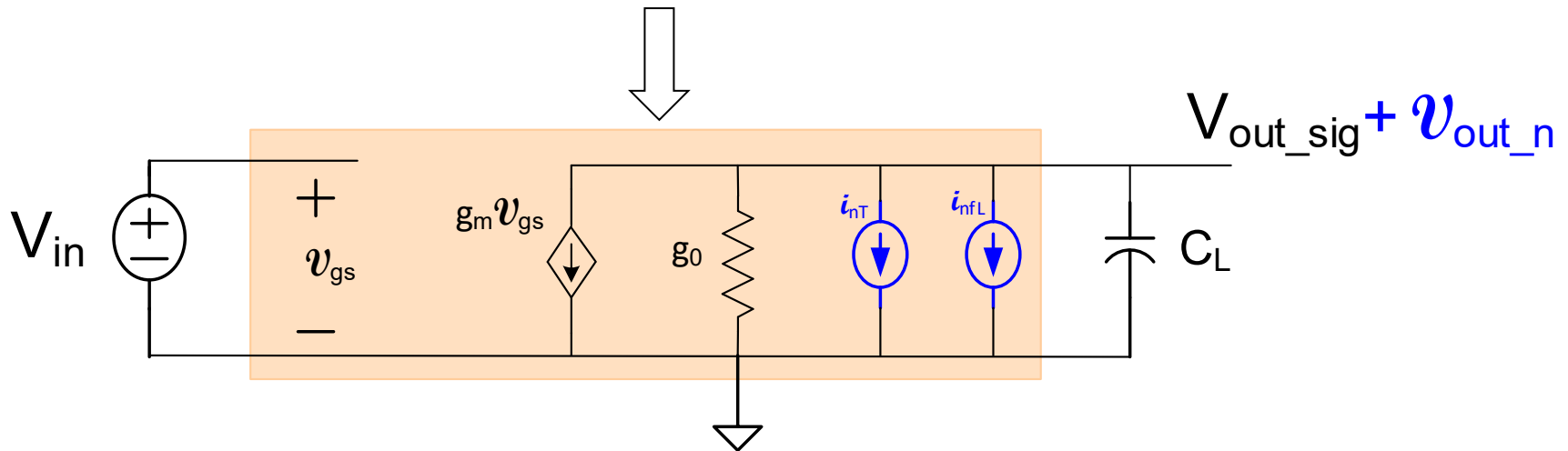
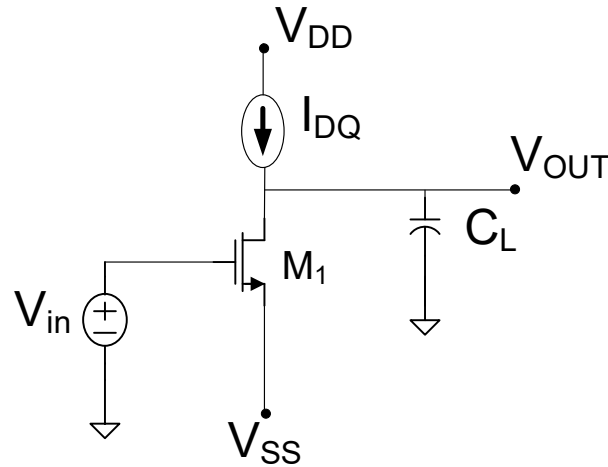
$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}} df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1 + \omega^2 R^2 C^2} df}$$

From a standard change of variable with a trig identity, it follows that

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}} df} = \sqrt{\frac{kT}{C}}$$

- The continuous-time noise voltage has an RMS value that is independent of R
- **Noise contributed by the resistor is dependent only upon the capacitor value C**
- This is often referred to as  $kT/C$  noise and it can be decreased at a given T only by increasing C

# Single-Stage Common-Source Op Amp



$$S_{OUT} = \left| \frac{1}{sC_L + g_O} \right|_{s=j\omega}^2 (S_T + S_{FL})$$

Where we are at:

# Basic Op Amp Design

- Fundamental Amplifier Design Issues

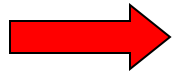
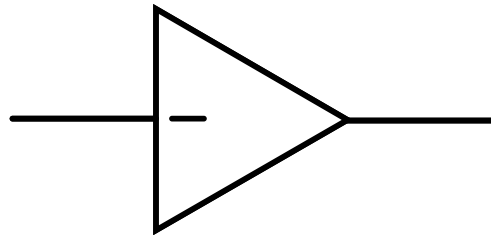


- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
- Other Basic Gain Enhancement Approaches

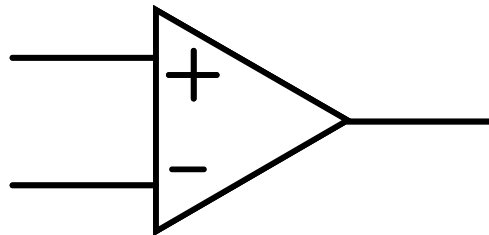
Where we are at:

# Single-Stage Low-Gain Op Amps

- Single-ended input



- Differential Input



**(Symbol does not distinguish between different amplifier types)**

# Differential Input Low Gain Op Amps

Will Next Show That :

- Differential input op amps can be readily obtained from single-ended op amps
- Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts



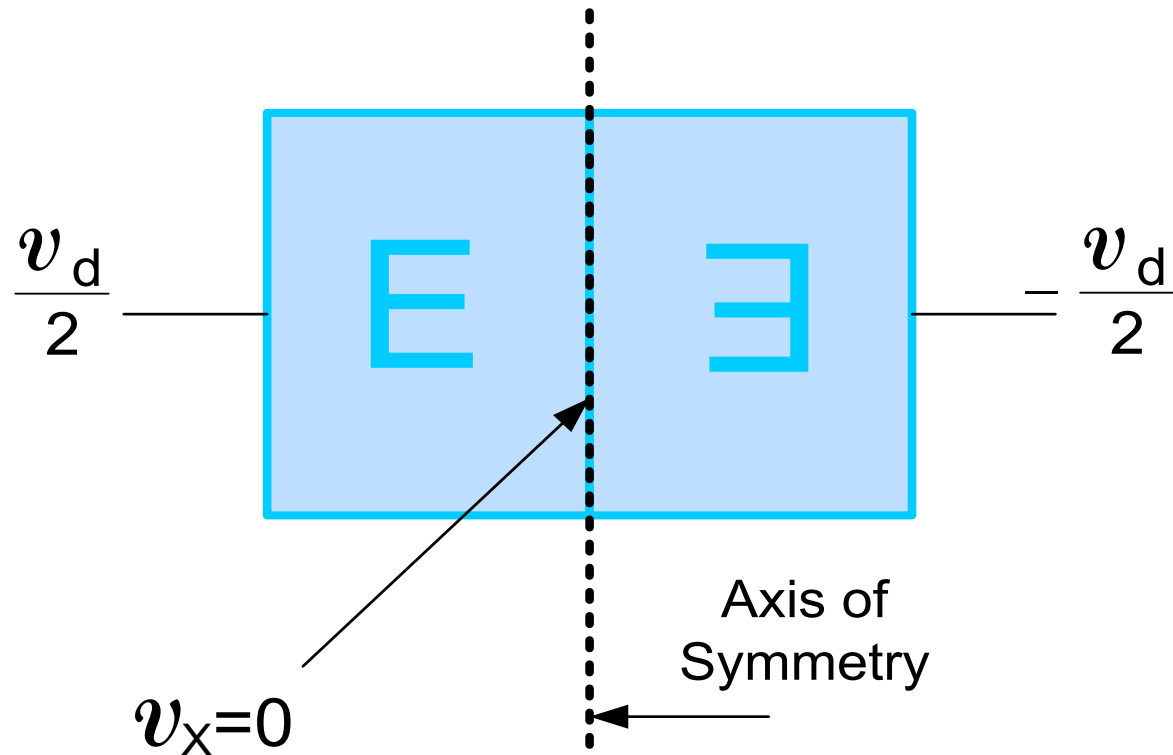
# Systematic strategies for designing and analyzing op amps

- Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used
- Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps
- Most authors present operational amplifiers from an “appear and analyze” approach

**A systematic strategy for designing and analyzing op amps will now be developed**

# Symmetric Networks

Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.

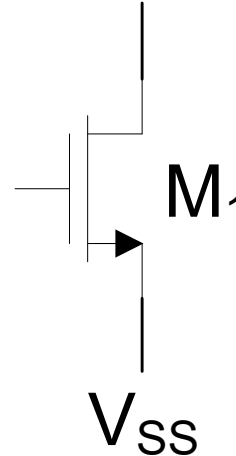
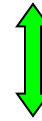
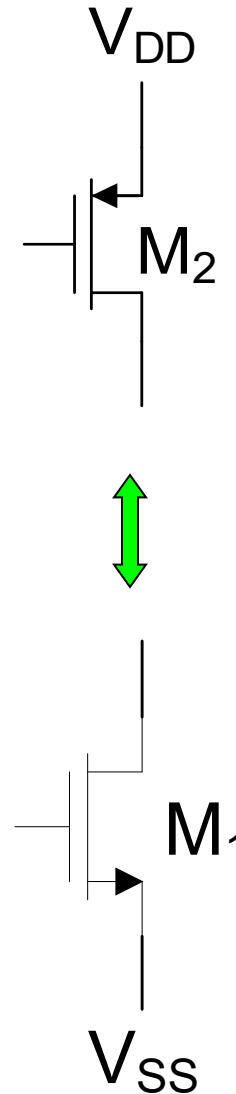
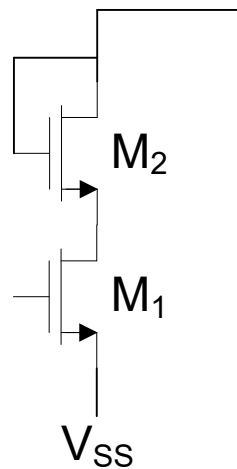
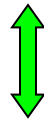
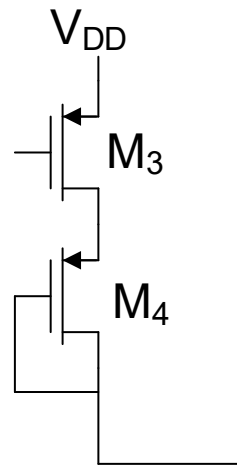


# Counterpart Networks

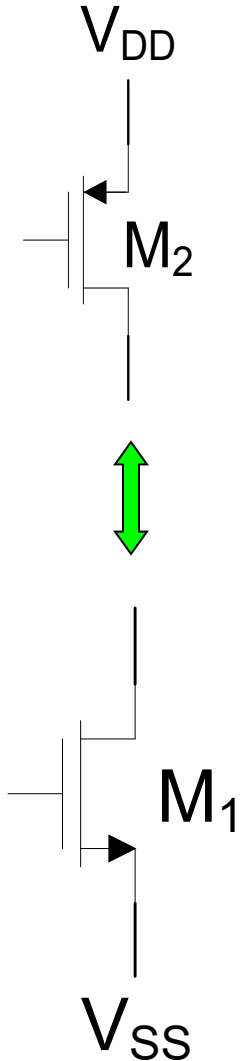
Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing  $V_{SS}$  biases with  $V_{DD}$  biases, and replacing all  $V_{DD}$  biases with  $V_{SS}$  biases.

# Counterpart Networks

Example:



# Counterpart Networks



the counterpart network is unique

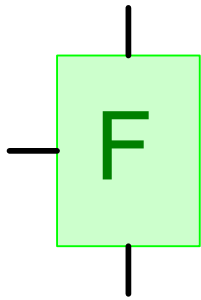
the counterpart of the counterpart is the original network

# Counterpart Networks

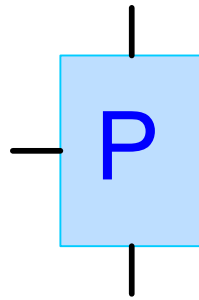
Theorem: The parametric expressions for all small-signal characteristics<sup>1</sup>, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.

<sup>1</sup>when expressed in the small-signal parameter domain

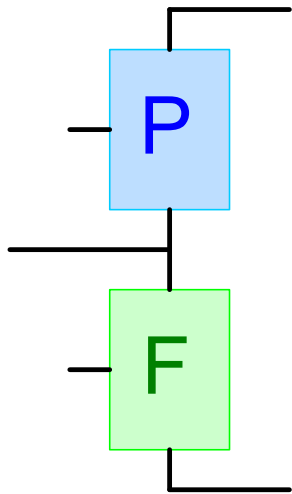
# Terminology and Notation



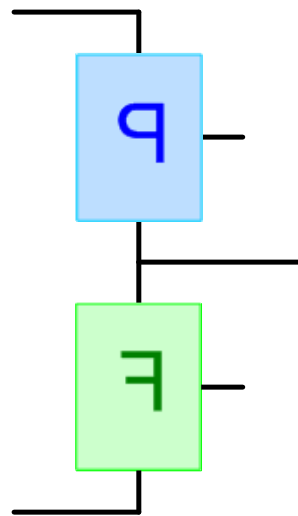
Quarter Circuit



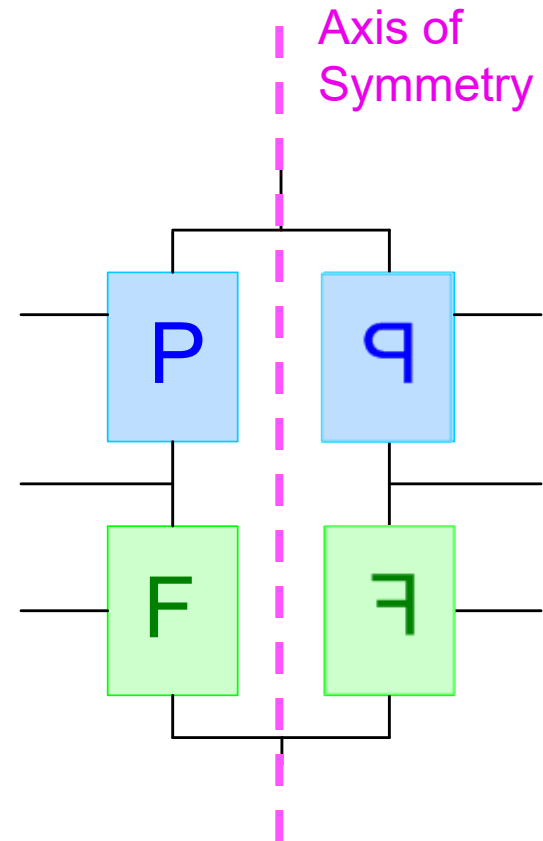
Counterpart Circuit



Half Circuit



“Reflected Image”  
Half Circuit

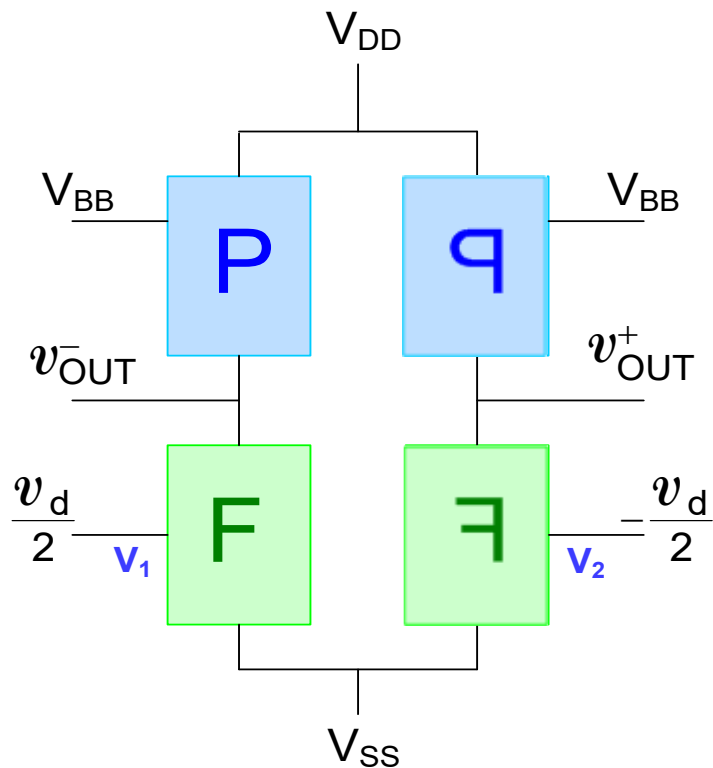


Symmetric  
Circuit

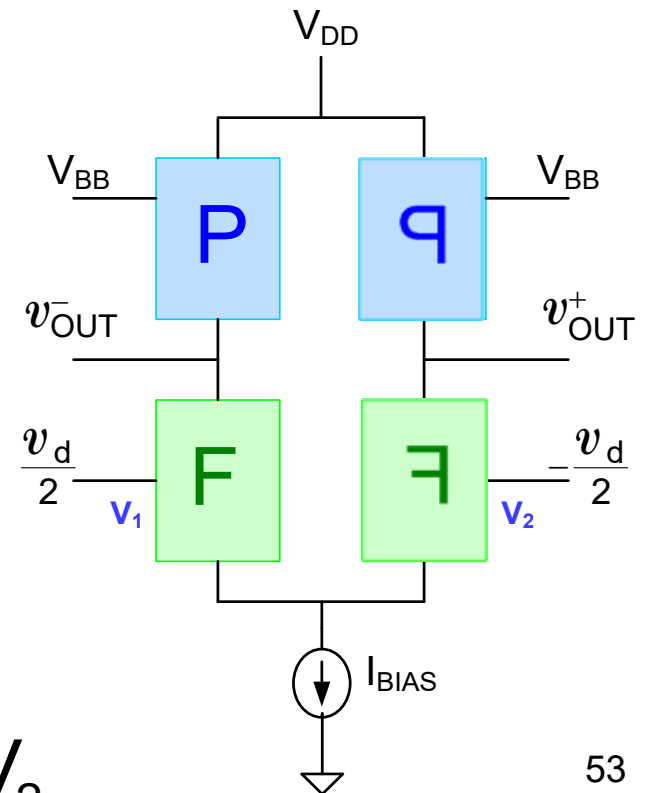
Reflected Image Half-circuit is identical to half-circuit, just drawn differently

# Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If  $F$  is any network with a single input and  $P$  is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

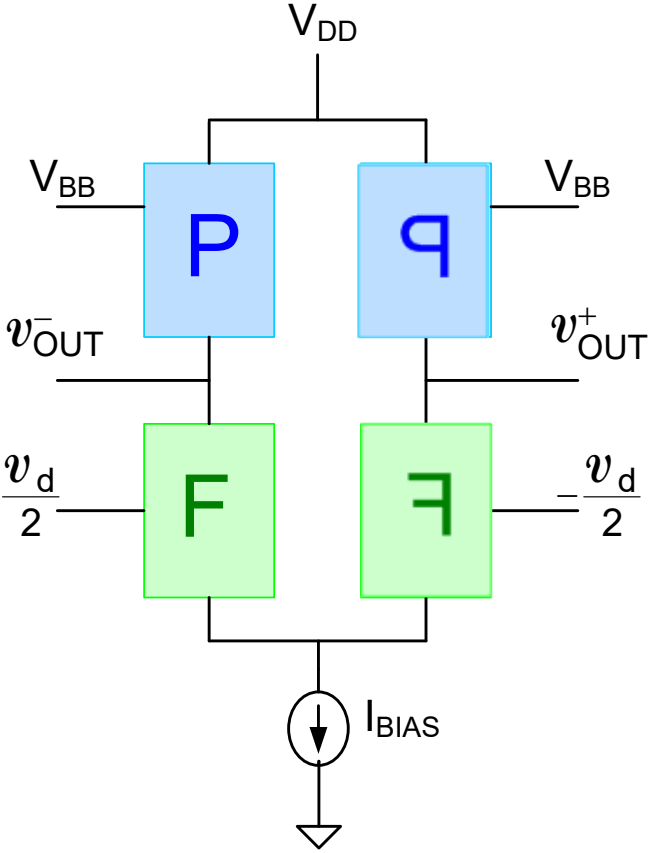


$$V_d = V_1 - V_2$$

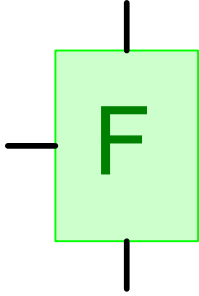




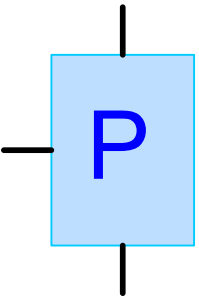
# Synthesis of fully-differential op amps from symmetric networks and counterpart networks



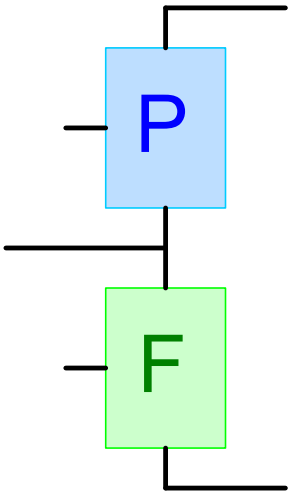
$$v_d = V_1 - V_2$$



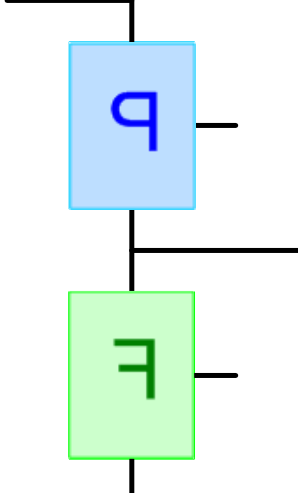
Quarter Circuit



Counterpart Circuit

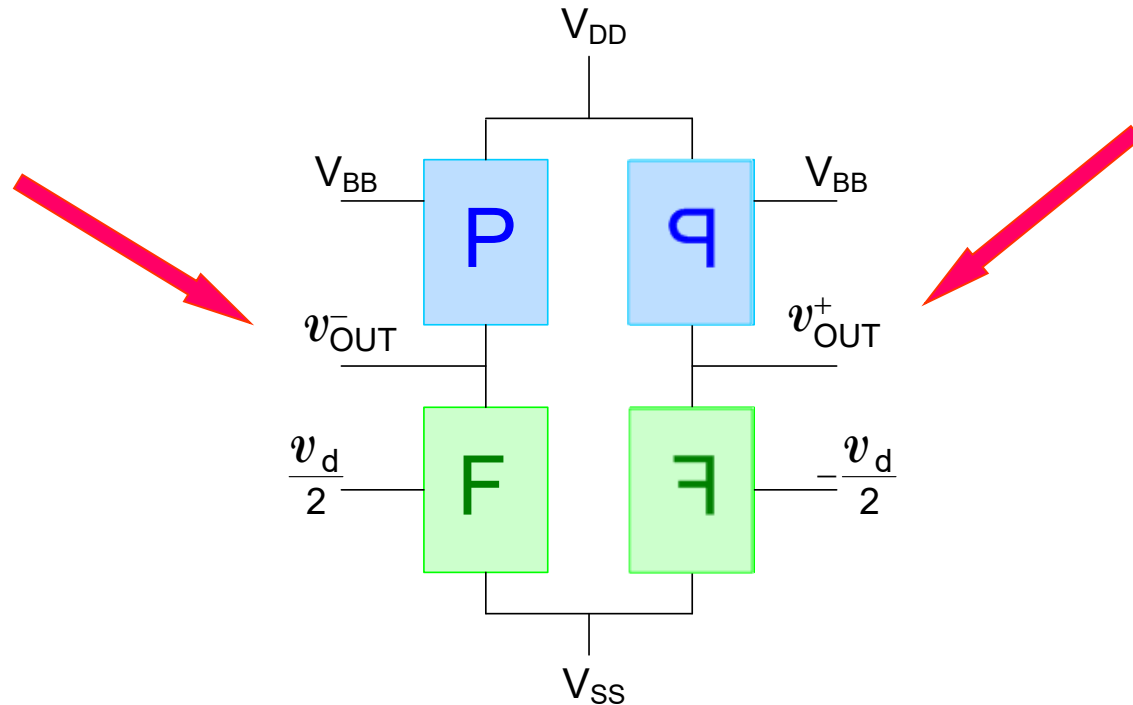


Half Circuit



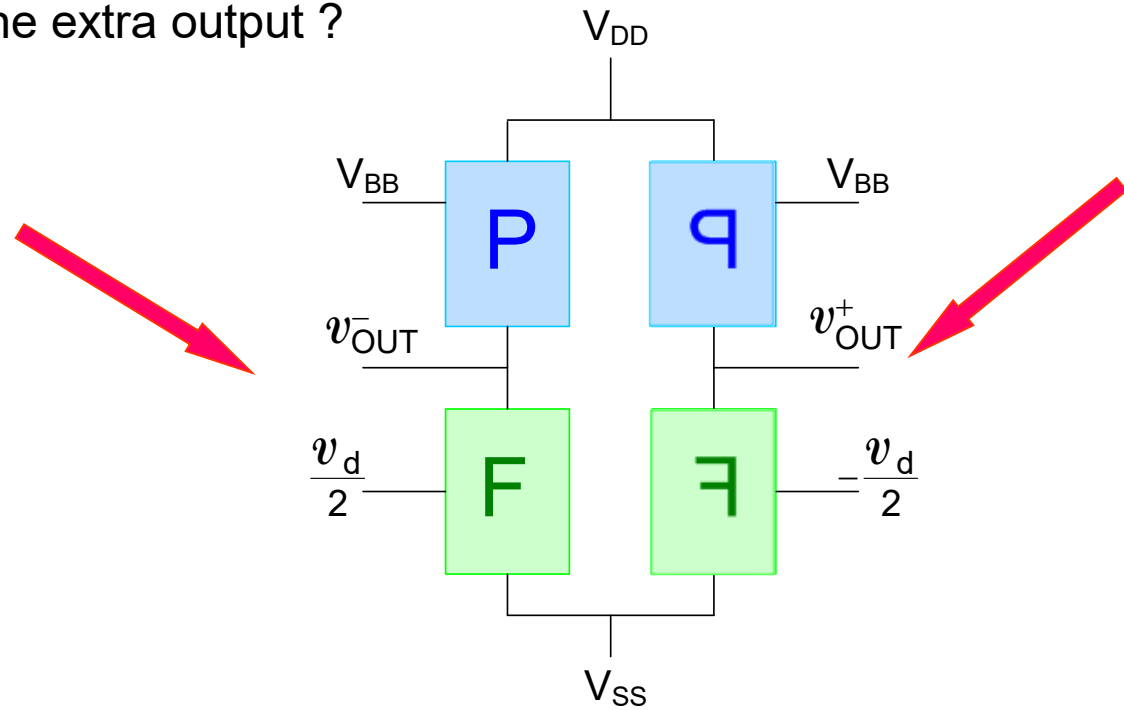
Reflected Image Half Circuit

# Synthesis of fully-differential op amps from symmetric networks and counterpart networks



What do we do with the extra output ?

What do we do with the extra output ?

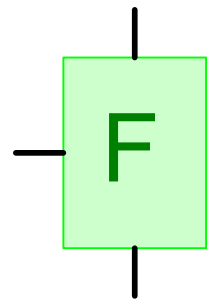


Use it or ignore it !!

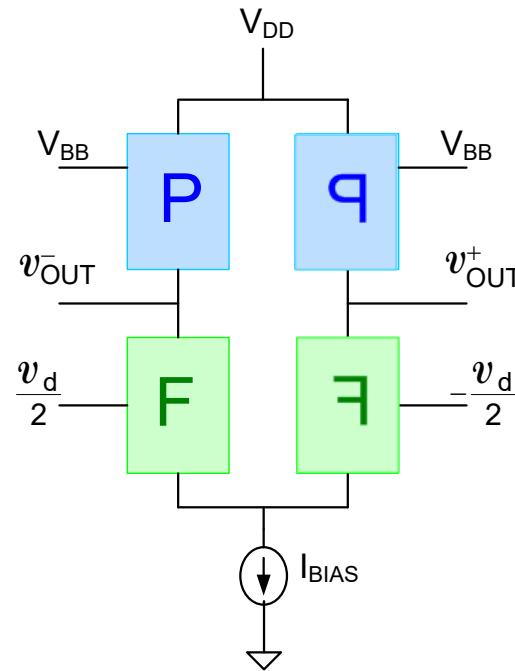


# Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.



Quarter Circuit



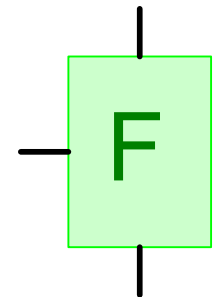
Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

# Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

**Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance**



**Quarter  
Circuit**



Stay Safe and Stay Healthy !

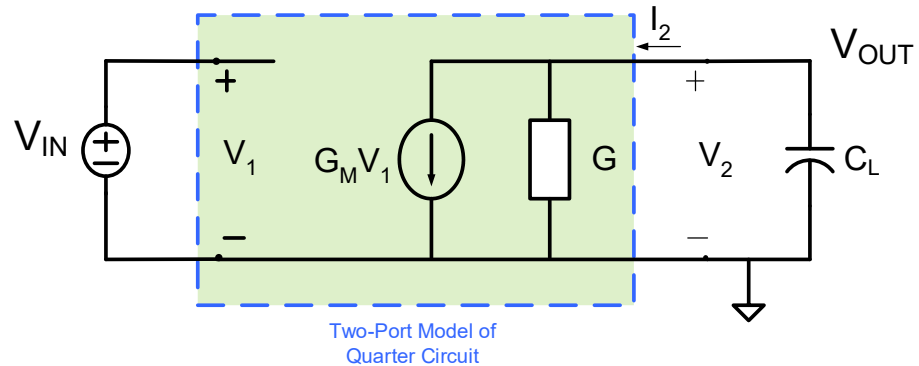
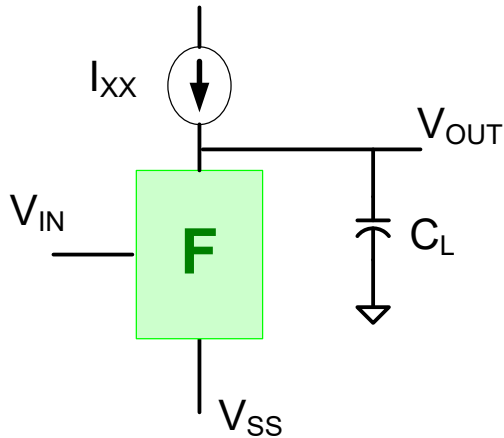
End of Lecture 3

- Fully Differential Single-Stage Amplifier
  - – General Differential Analysis
  - 5T Op Amp from simple quarter circuit
  - Biasing with CMFB circuit
  - Common-mode and differential-mode analysis
  - Common Mode Gain
  - Overall Transfer Characteristics
- Design of 5T Op Amp
- Slew Rate



# Characterization of Quarter Circuit

If the input impedance is infinite and circuit is unilateral, the two-port network only has two characterizing parameters :  $G_M$  and  $G$



$$\left. \begin{aligned} \mathbf{V}_{OUT}(\mathbf{G} + \mathbf{sC}_L) + \mathbf{G}_M \mathbf{V}_1 &= \mathbf{0} \\ \mathbf{V}_{IN} &= \mathbf{V}_1 \end{aligned} \right\}$$



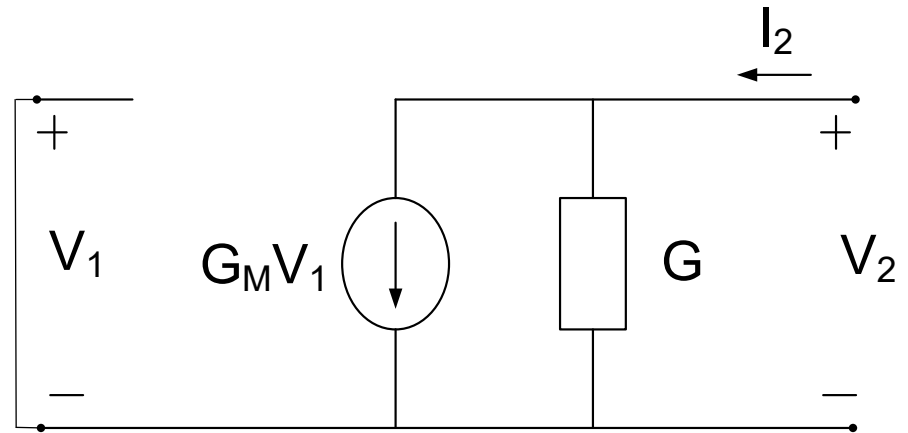
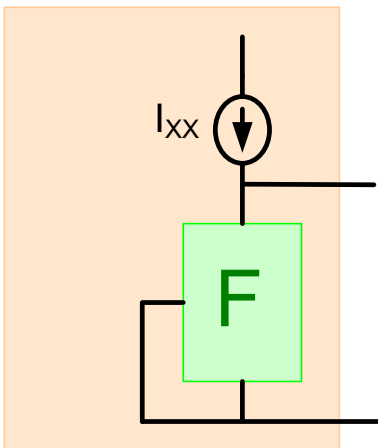
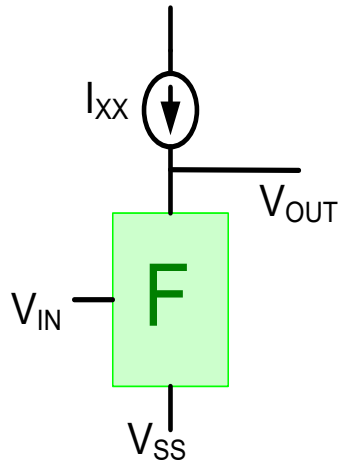
$$\mathbf{A}_{VQC}(\mathbf{s}) = \frac{-\mathbf{G}_M}{\mathbf{sC}_L + \mathbf{G}}$$

$$\mathbf{A}_{voqc} = -\frac{\mathbf{G}_M}{\mathbf{G}}$$

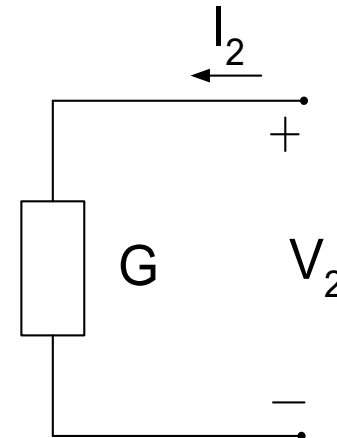
$$\mathbf{BW} = \frac{\mathbf{G}}{\mathbf{C}_L}$$

$$\mathbf{GB} = \frac{\mathbf{G}_M}{\mathbf{C}_L}$$

# Characterization of Quarter Circuit (or Counterpart Circuit) with input port terminated in small-signal short circuit



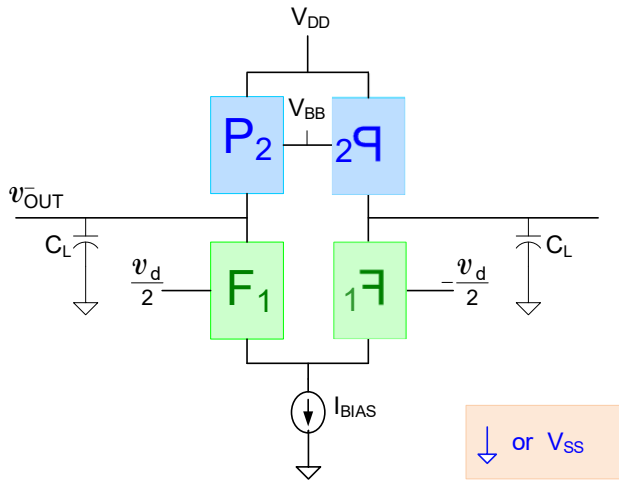
If the input port of a two-port has an ac short, then the two-port reduces to a one-port characterized by the conductance  $G$



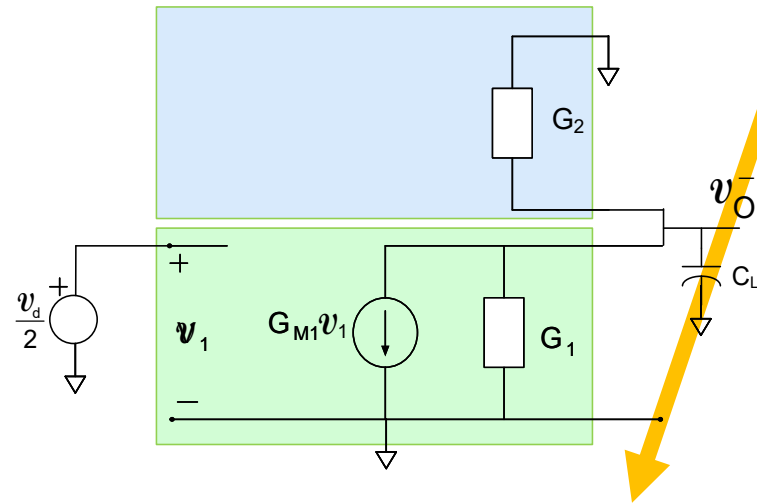
# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

$$A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$



Small signal differential half-circuit



Derivation:

from KCL and KVL:

$$\left. \begin{aligned} v_o^-(G_1 + G_2 + sC_L) + G_{M1}v_1 &= 0 \\ v_1 &= \frac{v_d}{2} \end{aligned} \right\} \Rightarrow$$

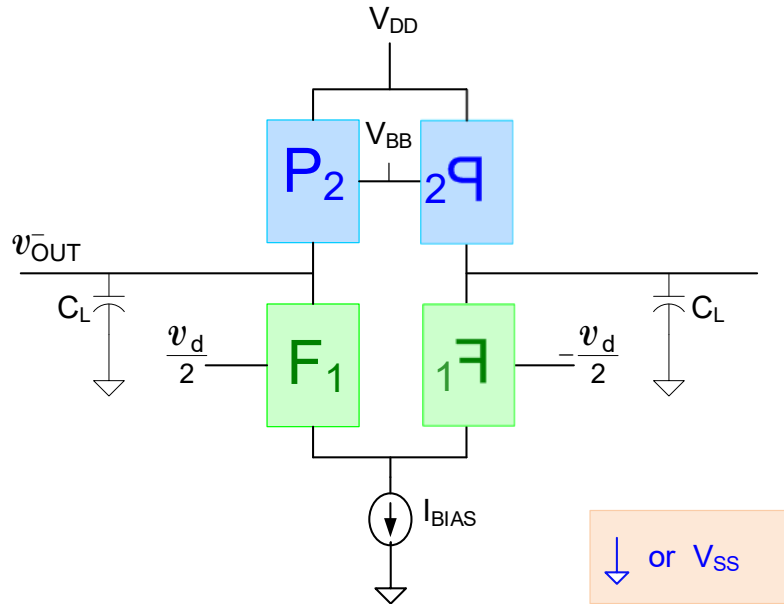
$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

Note: Factor of 2 reduction of differential gain since only half of the differential input is applied to the half-circuit

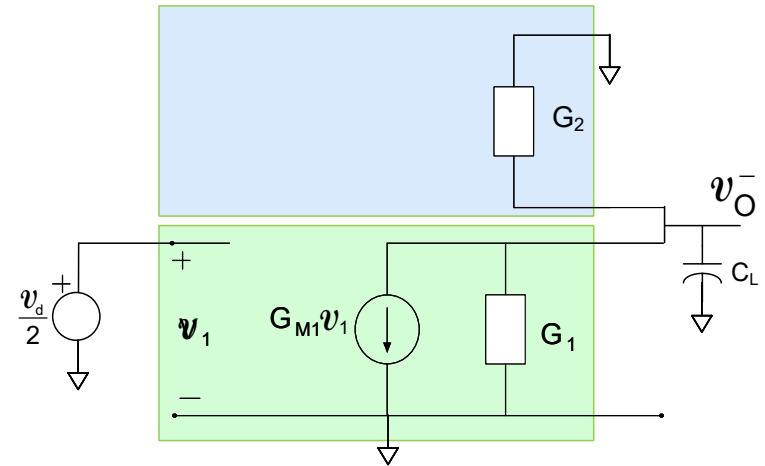
Note: More reduction of gain since denominator increases

# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --



Small signal differential half-circuit



$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$



$A_{V0}=?$   
 $BW=?$   
 $GB=?$

$$A_{V0} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

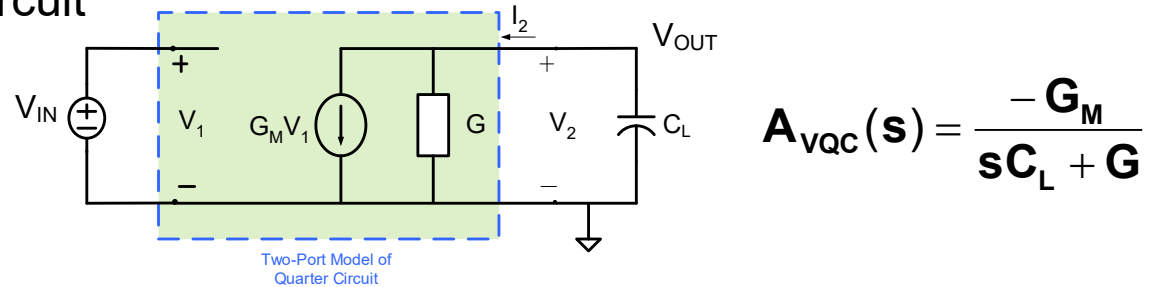
$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

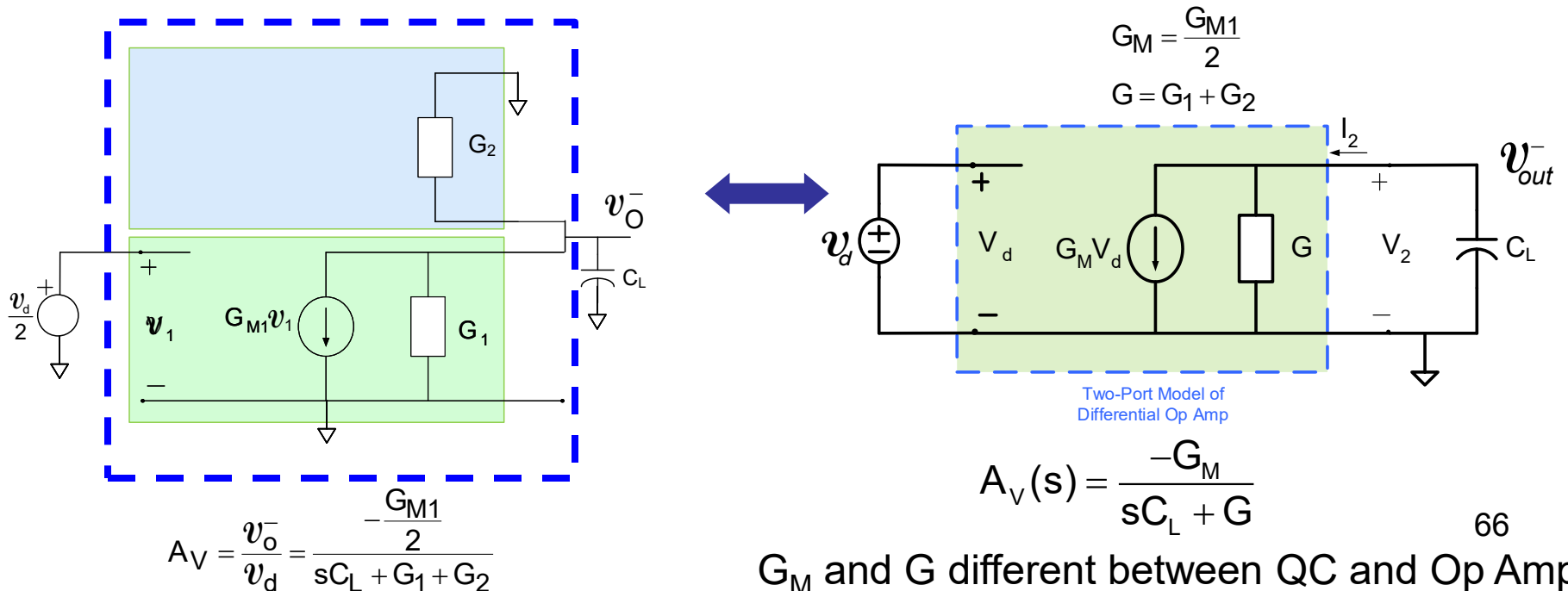
# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



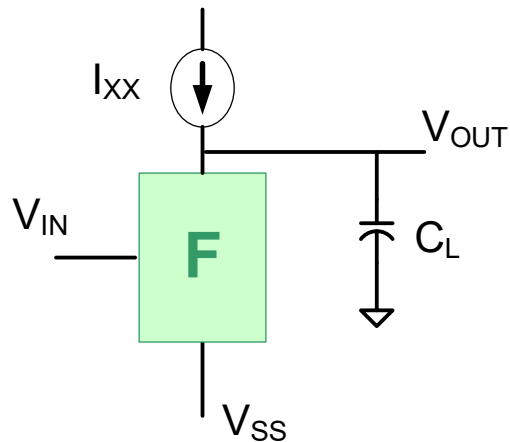
Small signal differential half-circuit (repeated from last slide) and Differential Op Amp



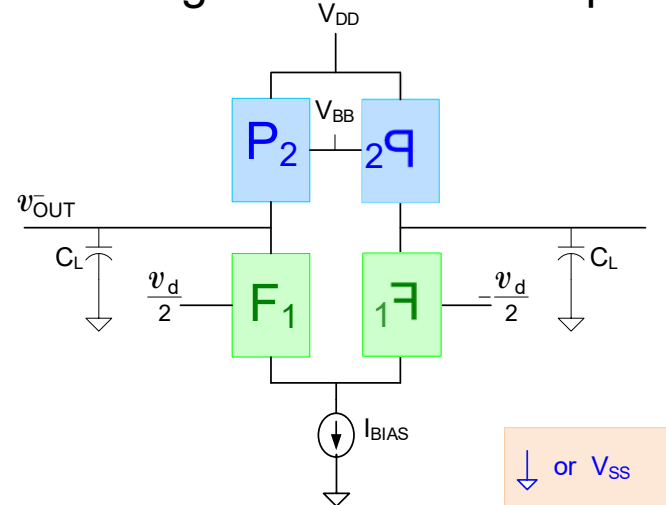
# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



Small signal differential amplifier



$$A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$

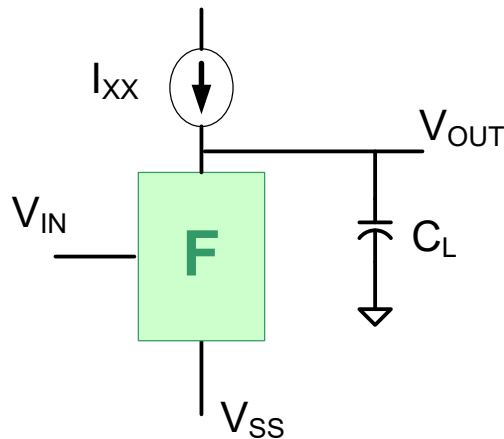


$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



$$A_{voqc} = -\frac{G_M}{G}$$

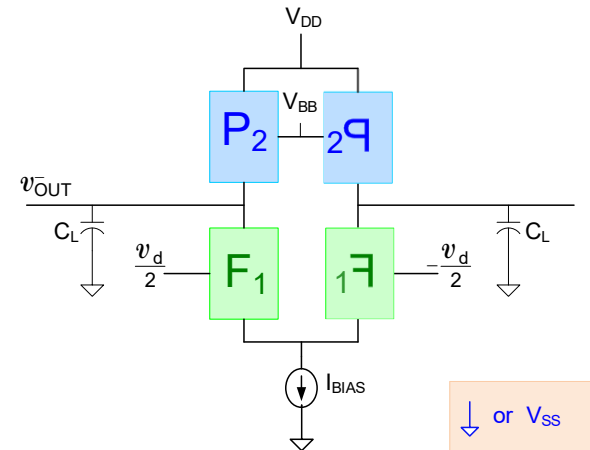
$$BW = \frac{G}{C_L} \quad GB = \frac{G_M}{C_L} \quad \longleftrightarrow$$

Note: Factor of 4 reduction of gain if  $G_1=G_2$  (this often occurs)

Note: Factor of 2 increase of BW if  $G_1=G_2$  (this often occurs)

Note: Factor of 2 reduction of GB if  $G_1=G_2$  (this often occurs)

Small signal differential amplifier



$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

$$A_{V0} = \frac{v_{OUT}^-}{v_d} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

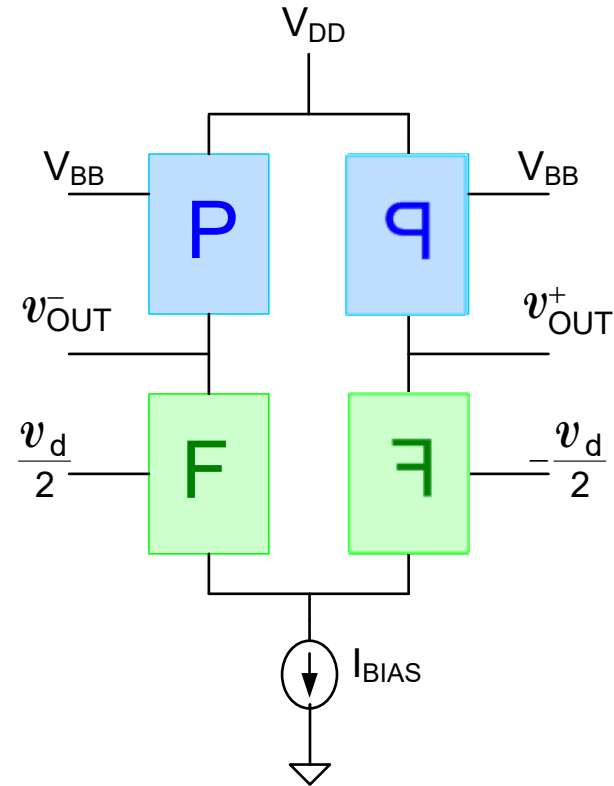
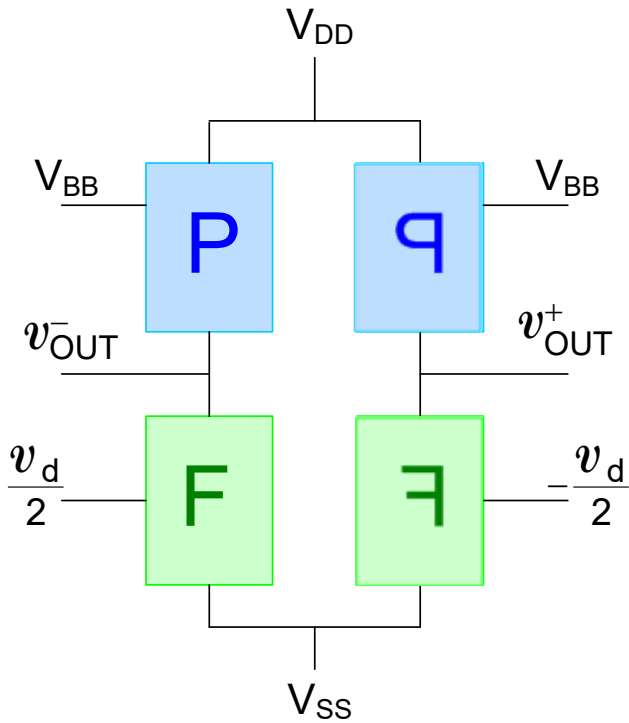
$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

Remember this is applicable to ANY quarter circuit !

# Comparison of Tail Voltage and Tail Current Source Structures

-- The "differential" gain --



Small signal half-circuits are identical so differential voltage gains, BW, and GB are all the same

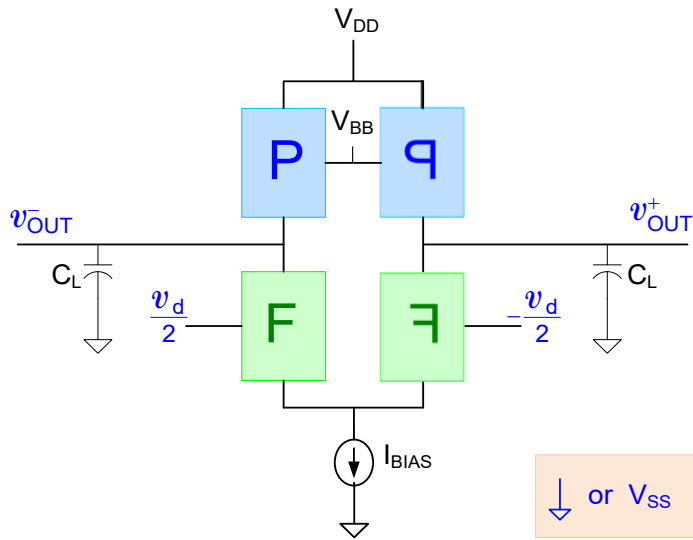
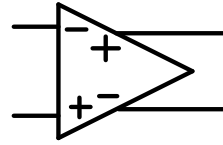


# Biasing Issues for Differential Amplifier

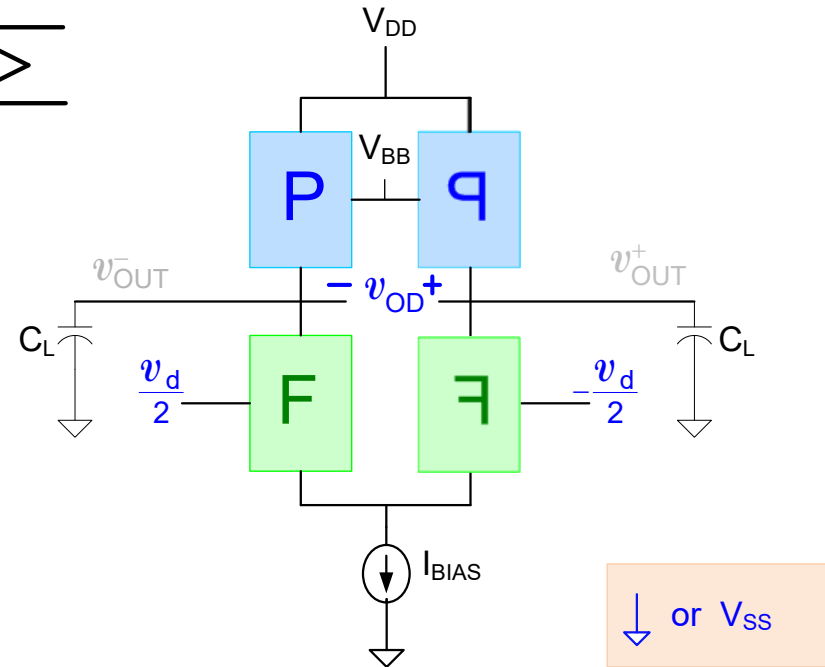
- Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing
- Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing

# Differential Output Amplifiers

-- The "differential" gain --




Single-Ended Outputs



Differential Output

Theorem: For a symmetric circuit with symmetric outputs and differential excitations:

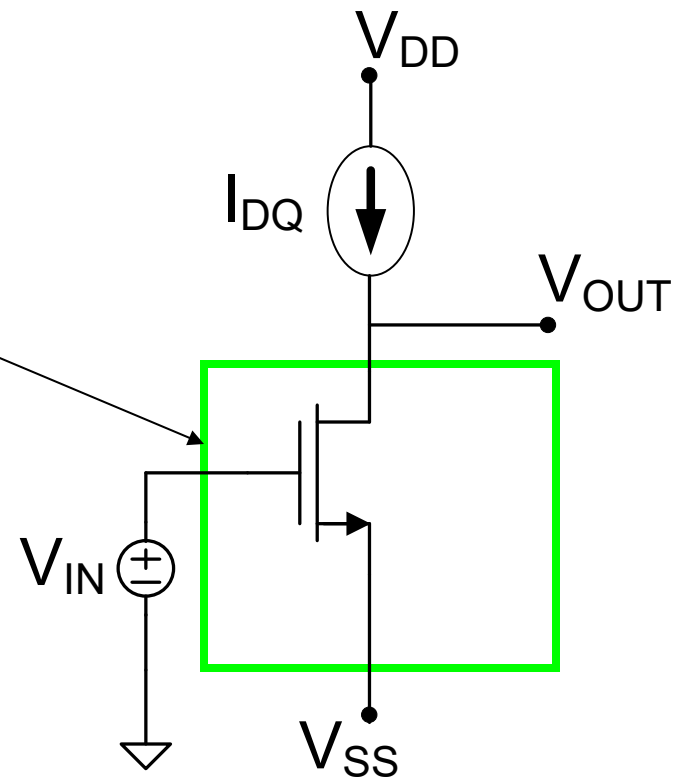
- Differential Voltage Gain Double that of Single-Ended Structure
- BW is the same
- GB Doubles for the Differential Output Structure

- Fully Differential Single-Stage Amplifier
  - General Differential Analysis
  -  – 5T Op Amp from simple quarter circuit
  - Biasing with CMFB circuit
  - Common-mode and differential-mode analysis
  - Common Mode Gain
  - Overall Transfer Characteristics
- Design of 5T Op Amp
- Slew Rate

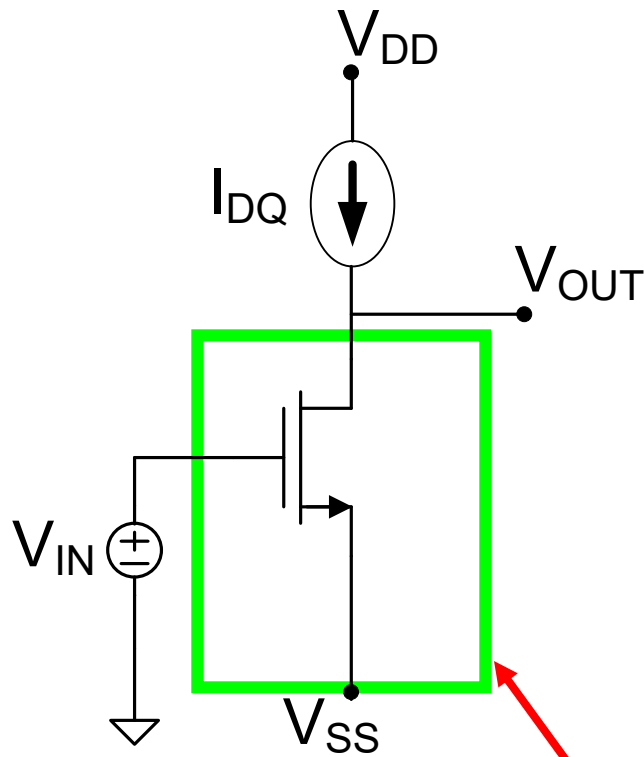
# Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier

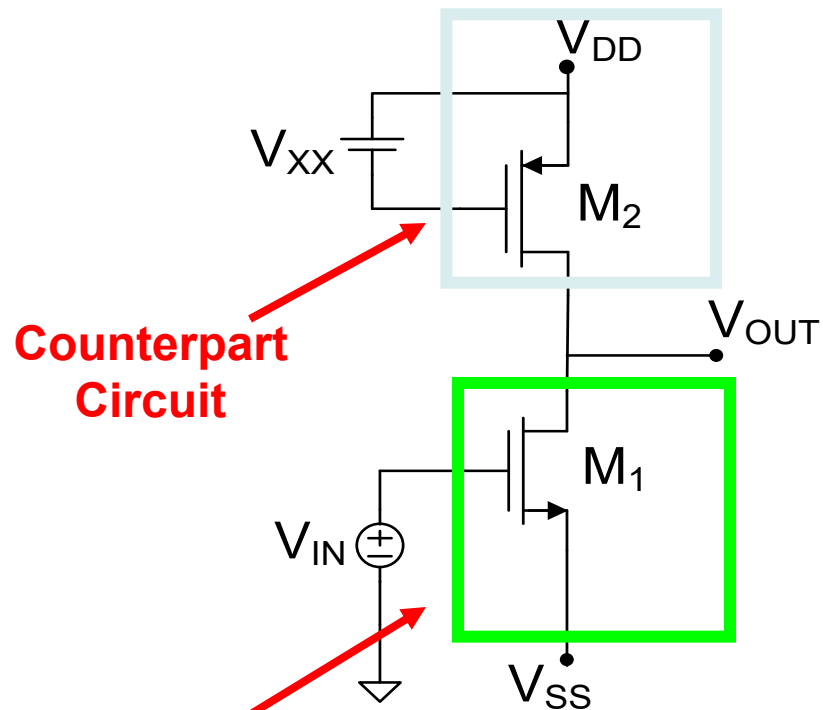
Quarter Circuit



# Single-stage single-input low-gain op amp



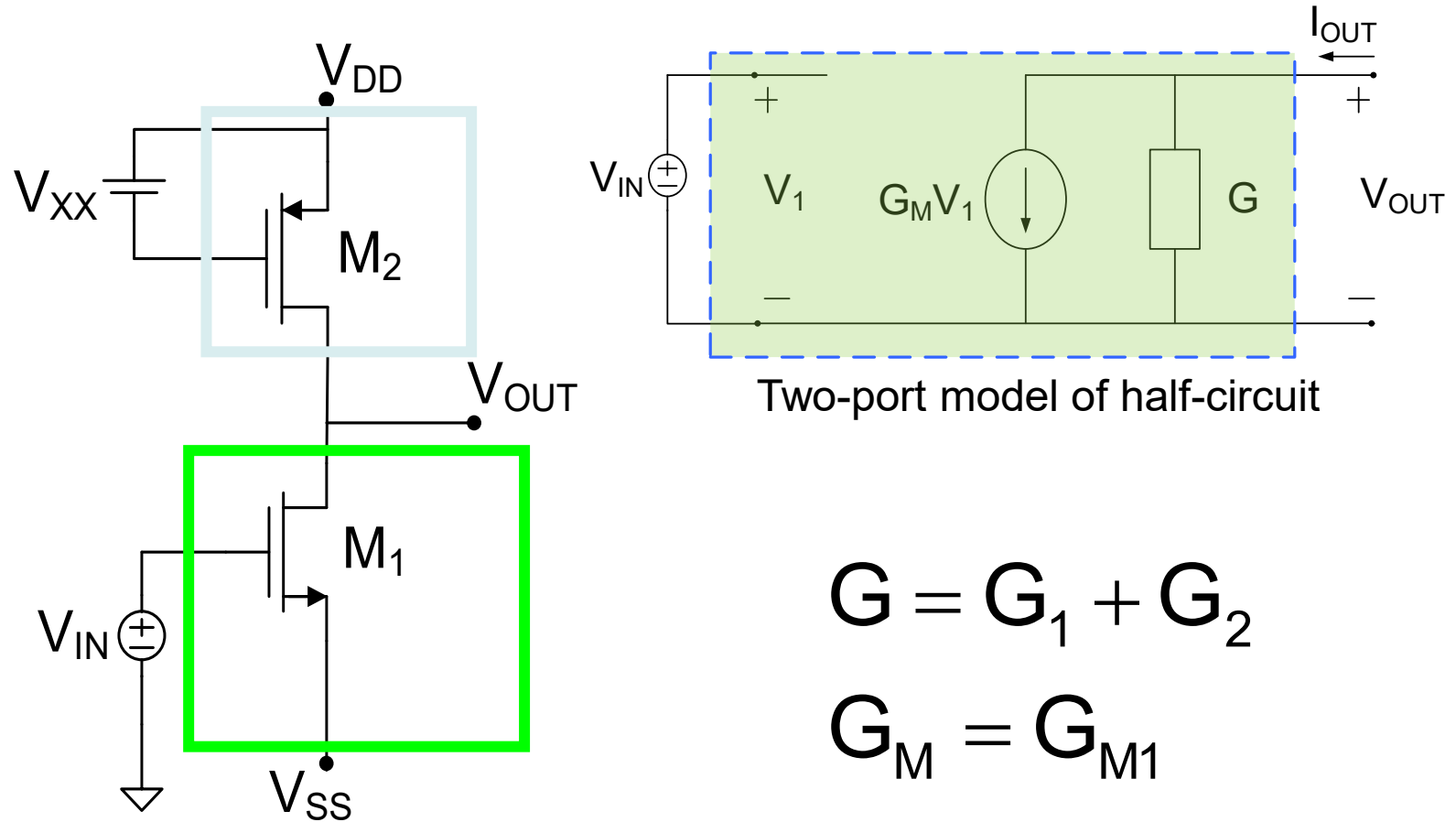
Basic Structure



Counterpart Circuit

Practical Implementation

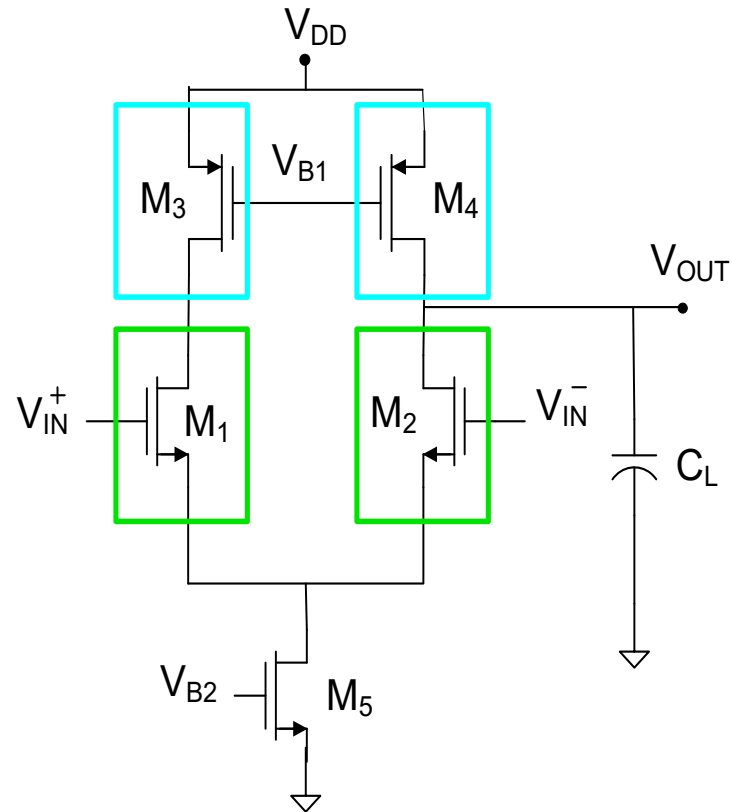
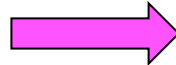
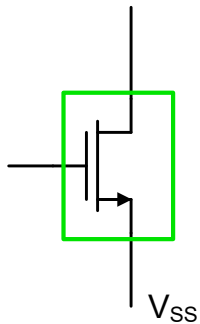
# Small signal model of half-circuit



# Single-stage low-gain differential op amp

-- The “differential” gain --

Single-Ended Output : Differential Input Gain



**Quarter Circuit**

Have synthesized fully differential op amp from quarter circuit !

**Termed the 5T Op Amp**

Will determine small-signal properties of 5T op amp by inspection (next time)

How many design variables?      How many Degrees of Freedom?



Stay Safe and Stay Healthy !



End of Lecture 3